

15.1 Maxwell waves, energy flow and boundary conditions

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS displacement current • Poynting vector

Objective

Use displacement current and material wave-speed relations; Determine wave-field directions and radiated energy flow; Apply Maxwell boundary conditions without confusing field components. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Vacuum Ampere–Maxwell law is $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(I + \epsilon_0 d\Phi E/dt)$. Displacement current depends on changing electric flux, not magnetic flux. It makes capacitor-loop results consistent for surfaces cutting the wire or crossing the gap. For a uniform gap field its value is $\epsilon_0 A dE/dt$.

In a homogeneous linear lossless medium, $v = 1/\sqrt{\mu\epsilon}$, $n = \sqrt{\mu\epsilon\epsilon_0}$. For nonmagnetic material, $v = c/\sqrt{\epsilon_r}$. Use constants appropriate to the frequency if dispersion matters. $E/B = v$ in this simple medium, and c in vacuum.

For propagation unit vector $\mathbf{n}$, $\mathbf{B} = (\mathbf{n} \times \mathbf{E})/v$. Phase $kz - \omega t$ travels $+z$; $kz + \omega t$ travels $-z$. A $+z$ wave with $\mathbf{E}$ along $x+y$ has $\mathbf{B}$ along $-x+y$. Poynting vector $\mathbf{E} \times \mathbf{B}/\mu$ gives energy-flow direction; far-zone radiation from an accelerating charge carries energy outward.

Because $\nabla \cdot \mathbf{B} = 0$, normal $\mathbf{B}$ is continuous across a boundary. Zero interior $\mathbf{B}$ in an ideal Meissner model therefore gives zero exterior normal $\mathbf{B}$; a tangential field can remain with a surface current. This conclusion uses the specified model, not an assumption of zero field everywhere. Normal $\mathbf{D}$ can instead jump by free surface charge.

■ Worked method

A plane wave travels along positive z in a linear lossless nonmagnetic dielectric. With relative permittivity 9,

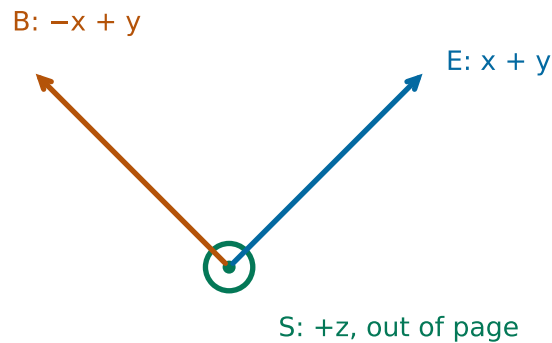
$$v = c/\sqrt{\epsilon_r} = (3.00 \times 10^8 \text{ m/s})/3 = 1.00 \times 10^8 \text{ m/s}.$$

If $\mathbf{E}$ points along positive x , $\mathbf{B} = \hat{\mathbf{z}} \times \mathbf{E}/v$ points along positive y . For $E_0 = 200 \text{ V/m}$,

$$B_0 = E_0/v = (200 \text{ V/m})/(1.00 \times 10^8 \text{ m/s}) = 2.00 \mu\text{T}.$$

$\mathbf{E} \times \mathbf{H}$ points along propagation. Check that material constants apply at the wave frequency.

$\mathbf{E} \times \mathbf{B}$ points in the propagation direction



2 Practice

2.1 Choose one answer. For a $+z$ plane wave whose electric field is along $+x$, magnetic field points: [1]

- **A** $+x$
- **B** $-x$
- **C** $+y$
- **D** $-y$
- **E** $+z$

2.2 Choose one answer. If interior $\mathbf{B}$ is zero at an ideal Meissner boundary, a nonzero outside $\mathbf{B}$ must be: [1]

- **A** Normal to the surface
- **B** Tangent to the surface
- **C** Zero at all outside points
- **D** Time dependent
- **E** Parallel to $\mathbf{E}$ in every case

Solutions

2.1 C $\mathbf{z} \times \mathbf{x} = \mathbf{y}$, ensuring $\mathbf{E} \times \mathbf{B}$ points $+\mathbf{z}$.

Compare the rejected options: $+\mathbf{x}$; $-\mathbf{x}$; $-\mathbf{y}$; $+\mathbf{z}$. Use the stated model and conditions.

2.2 B Normal $\mathbf{B}$ is continuous and therefore zero; a tangential component can remain.

Compare the rejected options: Normal to the surface; Zero at all outside points; Time dependent; Parallel to $\mathbf{E}$ in every case. Use the stated model and conditions.

2.3 0.25. $v/c = 1/\sqrt{16} = 1/4$.

2.4 True. The gap term is $\epsilon_0 d\Phi \mathbf{E}/dt$ in vacuum.

3.1 B $= \hat{\mathbf{n}} \times \mathbf{E}/c$ points along positive y since $(-\hat{x}) \times \hat{z} = \hat{y}$. $\langle S \rangle = E_0^2/(2\mu_0 c) = \epsilon_0 c E_0^2/2$ for a sinusoid. Absorbed power is $\langle S \rangle A$ and force magnitude is $\langle S \rangle A/c$, along negative x . Award magnetic direction, mean flux, power and force/direction. A perfect reflector would double the force; it is a different boundary condition.