

13.1 Electrostatic superposition, flux and conductors

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS solid angle • induced surface charge

Objective

Add Coulomb forces as vectors using the actual geometry; Compute flux from symmetry or solid angle with a stated orientation; Find conductor fields and potentials using induced surface charges. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Point-charge fields and Coulomb forces add as vectors. Same-direction components add, opposite components subtract, and perpendicular components combine by Pythagoras. Potential instead adds as a scalar, $V = \sum Q/(4\pi\epsilon_0 r)$, with a stated reference. Zero potential does not generally imply zero field: $E = -\nabla V$.

Gauss's law $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enclosed}}/\epsilon_0$ concerns a closed surface. Symmetry is needed to infer a simple local field. An infinite open plane subtends solid angle 2π , giving half a point charge's total flux in magnitude. Its sign depends on the normal; the plane does not enclose the charge.

For centred charge Q inside a conducting spherical shell of net charge q , equilibrium zero field in the material induces inner charge $-Q$ and outer charge $Q+q$. With outer radius b and zero potential at infinity, the material potential is $(Q+q)/(4\pi\epsilon_0 b)$, independent of observation radius.

In the cavity, a centred charge gives radial field $Q/(4\pi\epsilon_0 r^2)$. Potential includes constant shell contributions and is continuous across each surface; the normal field can jump. Off-centre Q still induces total inner charge $-Q$, but the cavity field is no longer the centred radial expression.

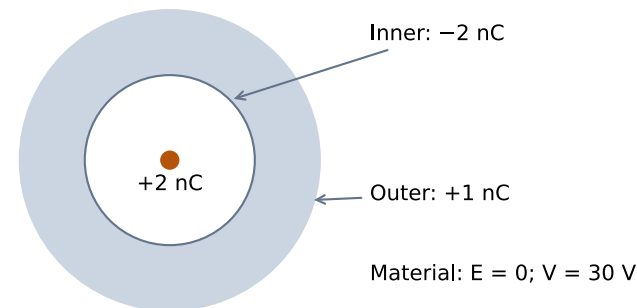
■ Worked method

A centred point charge Q lies in a conducting spherical shell of net charge q . Electrostatic equilibrium makes the field zero inside the metal. A Gaussian surface in the metal encloses zero net charge, so $Q_{\text{inner}} = -Q$. Charge conservation gives $Q_{\text{outer}} = q + Q$. With outer radius b and zero potential at infinity,

$$V_{\text{metal}} = \frac{q + Q}{4\pi\epsilon_0 b} = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(1.0 \times 10^{-9} \text{ C})}{0.30 \text{ m}} = 30 \text{ V}.$$

Here $Q = +2 \text{ nC}$ and $q = -1 \text{ nC}$. The cavity is not conducting material; its field need not vanish.

Shell net charge -1 nC ; outer radius 0.30 m



2 Practice

2.1 Choose one answer. A centred charge $+3Q$ lies inside a shell whose net charge is $-Q$. Its outer-surface charge is: [1]

- A $-3Q$
- B $-Q$
- C 0
- D $+2Q$
- E $+3Q$

2.2 Choose one answer. Two forces of magnitudes 6 N and 8 N are perpendicular. Their resultant magnitude is: [1]

- A 2 N
- B 7 N
- C 10 N
- D 14 N
- E 48 N

[2]

[2]

[5]

Solutions

2.1 D Inner surface has $-3Q$; shell total $-Q$ therefore leaves $+2Q$ outside.

Compare the rejected options: $-3Q$; $-Q$; 0 ; $+3Q$. Use the stated model and conditions.

2.2 C $\sqrt{6^2+8^2}=10$ N.

Compare the rejected options: 2 N; 7 N; 14 N; 48 N. Use the stated model and conditions.

2.3 0.5 . The plane subtends half the full solid angle.

2.4 True. The field is the negative potential gradient, not the potential itself.

3.1 The inner total remains $-Q$ because the metal field stays zero, but its distribution is generally nonuniform. Outer total remains $q+Q$. The spherical exterior is equipotential; uniqueness with zero at infinity gives $V = k(q + Q)/r$ outside, so metal potential and exterior field retain their centred values. Cavity field changes. Award inner total, redistribution, outer total, uniqueness argument and unchanged exterior/potential.