

13.1 Electrostatic superposition, flux and conductors

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS solid angle • induced surface charge

Objective

Add Coulomb forces as vectors using the actual geometry; Compute flux from symmetry or solid angle with a stated orientation; Find conductor fields and potentials using induced surface charges. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Point-charge fields and Coulomb forces add as vectors. Same-direction components add, opposite components subtract, and perpendicular components combine by Pythagoras. Potential instead adds as a scalar, $V = \Sigma Q/(4\pi\epsilon_0 r)$, with a stated reference. Zero potential does not generally imply zero field: $E = -\nabla V$.

Gauss's law $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enclosed}}/\epsilon_0$ concerns a closed surface. Symmetry is needed to infer a simple local field. An infinite open plane subtends solid angle 2π , giving half a point charge's total flux in magnitude. Its sign depends on the normal; the plane does not enclose the charge.

For centred charge Q inside a conducting spherical shell of net charge q , equilibrium zero field in the material induces inner charge $-Q$ and outer charge $Q+q$. With outer radius b and zero potential at infinity, the material potential is $(Q+q)/(4\pi\epsilon_0 b)$, independent of observation radius.

In the cavity, a centred charge gives radial field $Q/(4\pi\epsilon_0 r^2)$. Potential includes constant shell contributions and is continuous across each surface; the normal field can jump. Off-centre Q still induces total inner charge $-Q$, but the cavity field is no longer the centred radial expression.

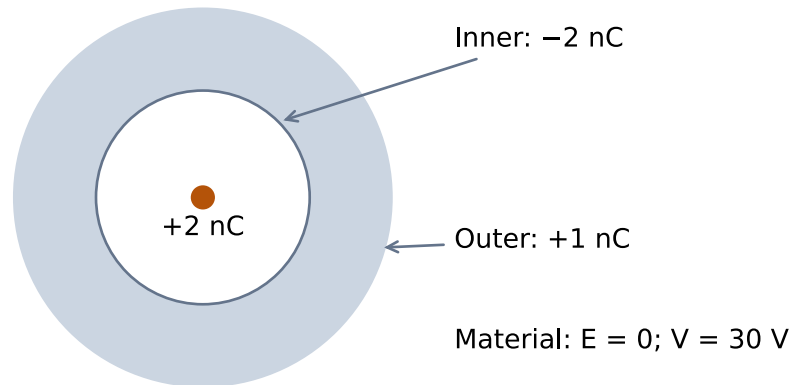
■ Worked method

A centred point charge Q lies in a conducting spherical shell of net charge q . Electrostatic equilibrium makes the field zero inside the metal. A Gaussian surface in the metal encloses zero net charge, so $Q_{\text{inner}} = -Q$. Charge conservation gives $Q_{\text{outer}} = q + Q$. With outer radius b and zero potential at infinity,

$$V_{\text{metal}} = \frac{q + Q}{4\pi\epsilon_0 b} = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(1.0 \times 10^{-9} \text{ C})}{0.30 \text{ m}} = 30 \text{ V}.$$

Here $Q = +2 \text{ nC}$ and $q = -1 \text{ nC}$. The cavity is not conducting material; its field need not vanish.

Shell net charge -1 nC ; outer radius 0.30 m



2 Practice

2.1 Choose one answer. A centred charge $+3Q$ lies inside a shell whose net charge is $-Q$. Its outer-surface charge is: [1]

- A $-3Q$
 - B $-Q$
 - C 0
 - D $+2Q$
 - E $+3Q$
-

2.2 Choose one answer. Two forces of magnitudes 6 N and 8 N are perpendicular. Their resultant magnitude is: [1]

- A 2 N
 - B 7 N
 - C 10 N
 - D 14 N
 - E 48 N
-

2.3 The magnitude of flux from a positive point charge through an infinite plane equals _____ times Q/ϵ_0 (decimal). Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: Inside equilibrium conducting material, nonzero constant potential is compatible with zero electric field. [2]

3 Exam-style questions

3.1 Move Q away from the centre while keeping it inside the closed cavity. The shell's exterior remains a sphere, with no external charges. Which of inner charge total, inner charge distribution, metal potential and exterior field retain their centred values? Explain using Gauss's law and electrostatic uniqueness. [5]

Solutions

2.1 D Inner surface has $-3Q$; shell total $-Q$ therefore leaves $+2Q$ outside.

Compare the rejected options: $-3Q$; $-Q$; 0 ; $+3Q$. Use the stated model and conditions.

2.2 C $\sqrt{6^2+8^2}=10$ N.

Compare the rejected options: 2 N; 7 N; 14 N; 48 N. Use the stated model and conditions.

2.3 0.5. The plane subtends half the full solid angle.

2.4 True. The field is the negative potential gradient, not the potential itself.

3.1 The inner total remains $-Q$ because the metal field stays zero, but its distribution is generally nonuniform. Outer total remains $q+Q$. The spherical exterior is equipotential; uniqueness with zero at infinity gives $V = k(q + Q)/r$ outside, so metal potential and exterior field retain their centred values. Cavity field changes. Award inner total, redistribution, outer total, uniqueness argument and unchanged exterior/potential.