

# 12.1 Coupled acceleration and relative motion

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 11 marks

KEY WORDS constraint • relative velocity

## Objective

Solve coupled-body equations using one acceleration constraint; Transform velocities between inertial frames; Check force and energy limits for constrained motion. Original public practice; not an ETS paper.

## 1 Key knowledge and worked examples

For table mass  $m_1$  and hanging mass  $m_2$  connected over an ideal pulley, a taut inextensible string gives one acceleration magnitude. Write  $T = m_1 a$  and  $m_2 g - T = m_2 a$ . Thus  $a = m_2 g / (m_1 + m_2)$ ,  $T = m_1 m_2 g / (m_1 + m_2) < m_2 g$ . Equating tension to weight incorrectly assumes no acceleration.

Equal tensions require an ideal massless string/pulley without friction or inertia. For pulley inertia  $I$ , use  $(T_2 - T_1)R = I\alpha$  and  $a = R\alpha$ . Draw separate bodies: a force internal to the combined system remains external to an individual body.

Galilean velocity transformation is  $v_{\text{relative}} = v_{\text{ground}} - V$ . A load released from a constant-speed level aircraft retains its horizontal ground velocity. With upward positive and no drag, vertical velocity is  $-gt$ . Relative horizontal velocity is zero; vertical displacement is  $-\frac{1}{2}gt^2$ , which is a different quantity.

Check limits:  $m_1 \rightarrow 0$  gives  $a \rightarrow g$  and  $T \rightarrow 0$ ; large  $m_1$  gives  $a \rightarrow 0$  and  $T \rightarrow m_2 g$  from below. Release-from-rest energy gives  $m_2 g s = \frac{1}{2}(m_1 + m_2)v^2$ , agreeing with  $v^2 = 2as$ . This check assumes no friction or other unaccounted work.

### ■ Worked method

A table mass  $m_1$  and hanging mass  $m_2$  share an acceleration through a taut ideal string. Choose rightward table motion and downward hanging motion positive.

$$F_T = m_1 a, \quad m_2 g - F_T = m_2 a.$$

$$a = \frac{m_2 g}{m_1 + m_2} = \frac{(2 \text{ kg})(10 \text{ m/s}^2)}{3 \text{ kg} + 2 \text{ kg}} = 4 \text{ m/s}^2.$$

$$F_T = m_1 a = (3 \text{ kg})(4 \text{ m/s}^2) = 12 \text{ N}.$$

$T$  is less than the hanging weight 20 N. A nonzero net downward force is needed for acceleration.

Subtract aircraft velocity; components in m/s

| Velocity after 3 s           | Horizontal | Vertical |
|------------------------------|------------|----------|
| Ground                       | 80         | -30      |
| Aircraft                     | 80         | 0        |
| Payload relative to aircraft | 0          | -30      |

## 2 Practice

**2.1** Choose one answer. For  $m_1=4$  kg on a frictionless table and  $m_2=1$  kg hanging,  $g=10$  m/s<sup>2</sup>, tension is: [1]

- **A** 10 N
- **B** 2 N
- **C** 8 N
- **D** 40 N
- **E** 0 N

**2.2** Choose one answer. A payload is dropped from a constant-speed level aircraft. With  $g=10$  m/s<sup>2</sup> and no drag, its velocity relative to that aircraft after 2 s is: [1]

- **A** 20 m/s downward
- **B** 20 m/s forward
- **C** 40 m/s downward
- **D** Zero
- **E** The aircraft speed forward plus 20 m/s downward

**2.3** For  $m_1=m_2$  and an ideal frictionless table/pulley,  $a/g=$ \_\_\_\_\_ (decimal). Show your relation and units. [2]



## Solutions

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**2.1 C**  $a=10/5=2 \text{ m/s}^2$  and  $T=m_1a=8 \text{ N}$ .

Compare the rejected options: 10 N; 2 N; 40 N; 0 N. Use the stated model and conditions.

**2.2 A** Horizontal velocities cancel; vertical relative speed is  $gt=20 \text{ m/s}$  downward.

Compare the rejected options: 20 m/s forward; 40 m/s downward; Zero; The aircraft speed forward plus 20 m/s downward. Use the stated model and conditions.

**2.3 0.5.**  $a=m_2g/(m_1+m_2)=g/2$ .

**2.4 True.** Net forces are  $m_1a$  and  $m_2a$ ; equal acceleration does not imply equal masses or forces.

**3.1**  $F_{T,1} = m_1a$ ;  $m_2g - F_{T,2} = m_2a$ ;  $(F_{T,2} - F_{T,1})R = I\alpha$ . With  $I = MR^2/2$  and  $\alpha = a/R$ ,  $F_{T,2} - F_{T,1} = Ma/2$ . Thus  $a = m_2g/(m_1 + m_2 + M/2)$ ,  $F_{T,1} = m_1m_2g/(m_1 + m_2 + M/2)$  and  $F_{T,2} = m_2(g - a)$ . At  $M = 0$  tensions become equal and the ideal result returns. Award body equations, torque/constraint, acceleration, tensions and limit.