11.1 Rotation, buoyancy and terminal-speed balances

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS moment of inertia • terminal speed

Objective

Use pivot torque and rotational inertia consistently; Partition rolling kinetic energy with the no-slip constraint ; Balance buoyancy and drag for composite bodies. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

About a fixed pivot, torque is r×F. A uniform rod pivoted at one end has I=ML^2/3; gravity acts at L/2. With angle θ from vertical, torque magnitude is Mg(L/2)sinθ, giving angular-acceleration magnitude 3g sinθ/(2L). Define direction separately; centre-of-mass inertia needs a parallel-axis correction.

No-slip rolling gives v=Rω. With I=βMR^2, K=½Mv^2(1+β) and the rotational fraction is β/(1+β). Disk: 1/3; sphere: 2/7; hoop: 1/2. Static friction can provide torque without dissipating energy at a stationary contact on a fixed surface.

Floating composites displace enough fluid to balance total weight. If submerged wood fraction is f with a stone dry, then putting stone volume Vs fully underwater reduces the wood fraction to f−Vs/V. The stone retains its weight while adding buoyancy; check that flotation/orientation assumptions are possible.

Terminal speed is a force balance. For quadratic drag Cv^2 and negligible buoyancy, vt=sqrt(Mg/C). Identical shapes share C, so a mass ratio r gives speed ratio sqrt(r). With buoyancy use effective weight; linear drag gives a different scaling. Terminal motion has nonzero balancing forces.

■ Worked method

A body rolls without slipping on a fixed surface. The no-slip constraint is v = R omega.

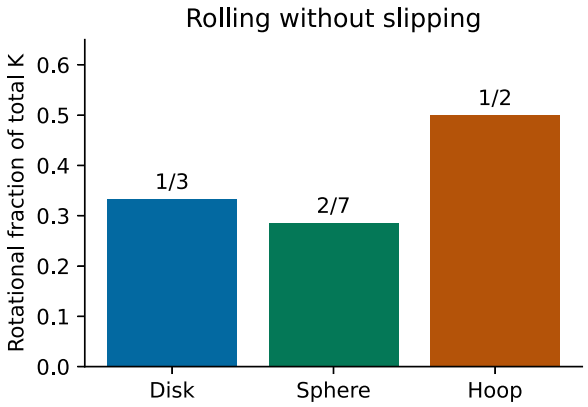
$$K = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}Mv^2(1+\beta), \qquad I = \beta MR^2.$$

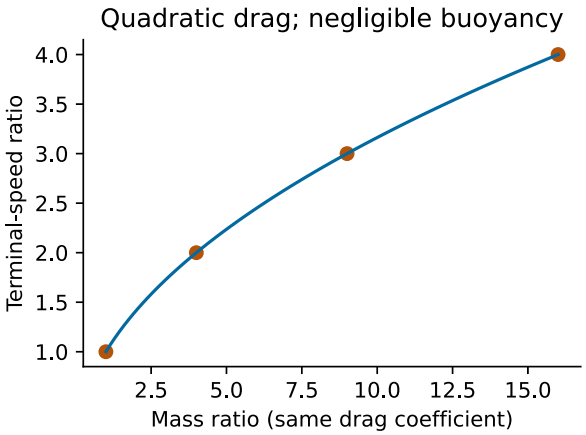
For a solid disk, beta = 1/2, so the rotational share is

$$\frac{K_{rot}}{K} = \frac{\beta}{1+\beta} = \frac{1/2}{3/2} = \frac{1}{3}.$$

For a uniform rod pivoted at one end, use $I = ML^2/3$, not its centre-of-mass inertia. About this pivot $\tau = -Mg(L/2)\sin\theta$, for theta measured from downward vertical.

$$\ddot{\theta} = \tau/I = -3g\sin\theta/(2L).$$





2 Practice

2.1 Choose one answer. For a uniform rod pivoted at one end, gravitational angular-acceleration magnitude at angle θ from vertical is: [1]

- **A** g sinθ/L
- **B** 3g sinθ/(2L)
- **C** 3g cosθ/(2L)
- **D** 6g sinθ/L
- **E** g/(2L)

2.2 Choose one answer. For a solid sphere I=2MR^2/5 rolling without slipping, the rotational fraction of total K is: [1]

- **A** 2/5
- **B** 1/2
- **C** 2/7
- **D** 5/7
- **E** 1/5

2.3 Identical-shape balls have quadratic drag and masses M and 9M, with negligible buoyancy. Their terminal-speed ratio is _____. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: At terminal speed the drag force vanishes. [2]

3 Exam-style questions

3.1 A solid sphere rolls from rest down a vertical height h without slipping or energy loss. Use $I = 2MR^2/5$ to find its bottom speed. Compare with a frictionless sliding point mass. State which assumption would fail if the sphere slipped. [4]

Solutions

2.1 B Divide Mg(L/2)sinθ by $ML^2/3$.

Compare the rejected options: g sinθ/L; 3g cosθ/(2L); 6g sinθ/L; g/(2L). Use the stated model and conditions.

2.2 C Use $\beta/(1+\beta)=(2/5)/(7/5)=2/7$.

Compare the rejected options: 2/5; 1/2; 5/7; 1/5. Use the stated model and conditions.

2.3 3. The ratio is sqrt(9).

2.4 False. Drag balances the effective weight; it is generally nonzero.

3.1 Energy gives $Mgh = \frac{1}{2}Mv^2(1+2/5)$, hence $v = \sqrt{10gh/7}$. The slider has $v_s = \sqrt{2gh}$, so $v/v_s = \sqrt{5/7} < 1$. Slipping breaks $v = R\omega$ and can dissipate energy through kinetic friction. Award energy relation, speed, comparison and the constraint/loss distinction.