

10.1 Collisions, work and oscillator energy

Name: _____ Class: _____ Date: _____

Total: 11 marks**KEY WORDS** external impulse • turning point

Objective

Compute vector momentum and energy loss in sticking collisions; Compare fixed-force springs and use work–energy conditions; Derive small-oscillation periods and oscillator energies. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

With negligible external impulse, conserve momentum in each direction. For sticking masses, $\mathbf{v} = (m_1\mathbf{v}_1 + m_2\mathbf{v}_2)/(m_1 + m_2)$. Add vectors before taking magnitudes. Compare $\Sigma \frac{1}{2}m|\mathbf{v}|^2$ initially with $\frac{1}{2}(m_1 + m_2)|\mathbf{v}|^2$ finally; the loss becomes internal energy/deformation. Sticking does not mean final rest.

Work–energy gives $\Delta K = \int \mathbf{F} \cdot d\mathbf{r}$, or Fs for constant parallel force. Mechanical-energy conservation requires no unaccounted nonconservative work. Spring energy is $\frac{1}{2}kx^2$. At fixed force, $x = F/k$ and $U = F^2/(2k)$; at fixed extension U increases with k . These comparisons have opposite answers.

Near stable equilibrium, $U \approx U_0 + \frac{1}{2}k_{\text{eff}}(x - x_0)^2$ with $k_{\text{eff}} = U''(x_0) > 0$. Then $\omega = \sqrt{k_{\text{eff}}/m}$, $T = 2\pi/\omega$. A small-angle pendulum uses $\sin\theta \approx \theta$ in $\theta\ddot{\theta} + (g/L)\sin\theta = 0$, giving $T = 2\pi\sqrt{L/g}$. Large amplitudes need a correction; ideal pendulum mass cancels.

Undamped oscillator energy is $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$. At equilibrium it is all kinetic; at turning points it is all potential. Use $\omega = 2\pi f$ and $A = v_{\text{max}}/\omega$. The measurement position determines which energy expression is simplest; convert joules to millijoules only after calculation.

■ Worked method

Two carts stick with negligible external impulse. Add momentum vectors, then divide by total mass. Known: $m_1 = 2 \text{ kg}$, $\mathbf{v}_1 = (3, 0) \text{ m/s}$; $m_2 = 1 \text{ kg}$, $\mathbf{v}_2 = (0, 3) \text{ m/s}$.

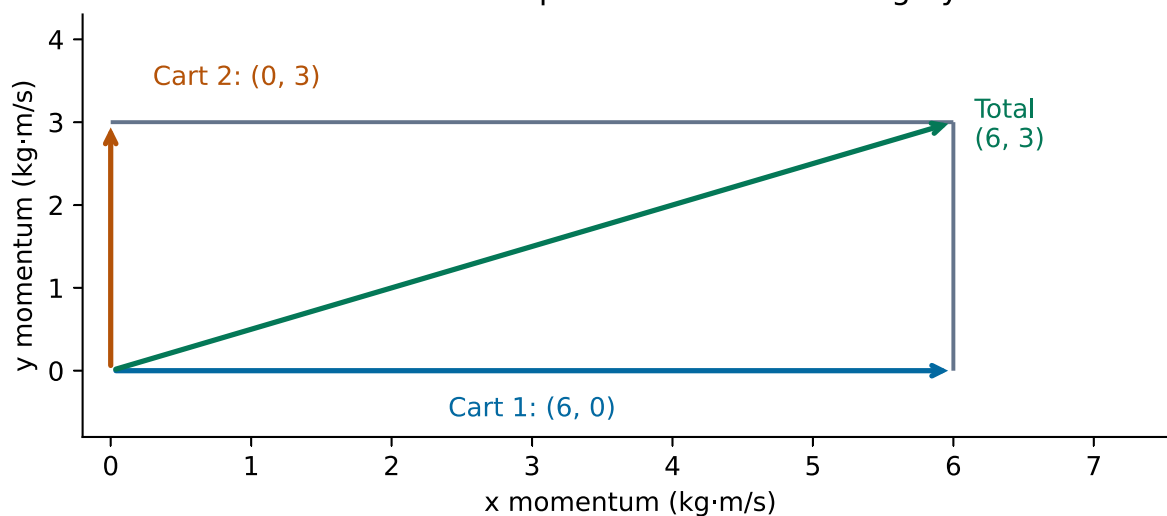
$$\mathbf{v}_f = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2} = \frac{(2 \text{ kg})(3, 0) \text{ m/s} + (1 \text{ kg})(0, 3) \text{ m/s}}{3 \text{ kg}} = (2, 1) \text{ m/s}.$$

$$K_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(2 \text{ kg})(3 \text{ m/s})^2 + \frac{1}{2}(1 \text{ kg})(3 \text{ m/s})^2 = 13.5 \text{ J}.$$

$$K_f = \frac{1}{2}(m_1 + m_2)|\mathbf{v}_f|^2 = \frac{1}{2}(3 \text{ kg})(5 \text{ m}^2/\text{s}^2) = 7.5 \text{ J}.$$

The 6 J difference becomes internal energy. Kinetic energy is not conserved in sticking.

Add momentum components before dividing by mass



2 Practice

2.1 Choose one answer. For the two carts in the worked example, the common velocity is: [1]

- A (3,3) m/s
- B (2,1) m/s
- C (1,2) m/s
- D (2,2) m/s
- E (0,0) m/s

2.2 Choose one answer. Two springs with $k_1=3k_2$ hold the same force. The ratio U_1/U_2 is: [1]

- A 3
- B 9
- C 1
- D $1/3$
- E $1/9$

Solutions

2.1 B Divide the vector momentum (6,3) by total mass 3, component by component.

Compare the rejected options: (3,3) m/s; (1,2) m/s; (2,2) m/s; (0,0) m/s. Use the stated model and conditions.

2.2 D At fixed force $U = F^2/(2k)$, hence the inverse stiffness ratio.

Compare the rejected options: 3; 9; 1; 1/9. Use the stated model and conditions.

2.3 3. T scales as $\sqrt{L}$.

2.4 True. Displacement is zero, so spring energy is zero; speed is maximal.

3.1 The sum of equal velocity vectors has magnitude $2u|\cos(\theta/2)|$. Dividing momentum by total mass gives $v_f = u|\cos(\theta/2)|$. $K_i = mu^2$ and $K_f = mv_f^2$, so lost fraction is $\sin^2(\theta/2)$. The three limits are 0, 1/2 and 1. Award vector sum, speed, energy relation, lost fraction and limits. The formula uses $0 \leq \theta \leq \pi$ between directions.