

# 1.1 Classical mechanics

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 12 marks

KEY WORDS Lagrangian • angular momentum

## Objective

Apply Newtonian, Lagrangian and Hamiltonian descriptions; Analyse rotation, central forces and oscillations; Use conservation laws with their conditions. Original public practice; not an ETS paper.

## 1 Key knowledge and worked examples

Newton's second law is a vector statement. Choose coordinates and identify constraints before components.

Energy conservation applies when work from nonconservative forces is accounted for; momentum conservation needs zero net external impulse.

For small oscillations, linearise around a stable equilibrium. A restoring term proportional to displacement gives harmonic motion.

Lagrange's equations use  $L=T-V$  and generalised coordinates. This undergraduate formalism is distinct from school-level force substitution.

### ■ Worked method

Choose an inertial frame and a coordinate  $x$  measured from spring equilibrium. For a conservative spring, the Lagrangian avoids solving constraint forces.

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2.$$

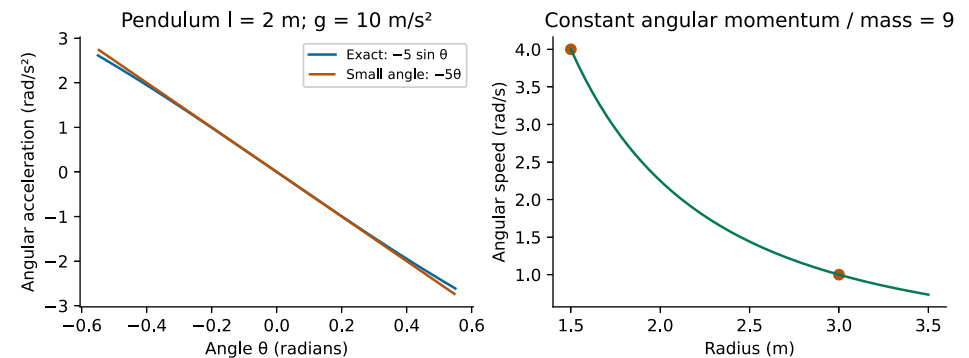
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0, \quad m\ddot{x} + kx = 0.$$

With  $m = 2.0$  kg and  $k = 18$  N/m,

$$\omega = \sqrt{k/m} = \sqrt{(18 \text{ N/m})/(2.0 \text{ kg})} = 3.0 \text{ rad/s}.$$

$$T = 2\pi/\omega = 2\pi/(3.0 \text{ rad/s}) = 2.09 \text{ s}.$$

Positive stiffness gives stable motion. The units of  $k/m$  are inverse seconds squared.



## 2 Practice

2.1 Choose one answer. Doubling spring mass changes angular frequency by: [1]

- A 2
- B  $1/\sqrt{2}$
- C  $1/2$

2.2 Choose one answer. In an isolated perfectly inelastic collision, conserved quantity is: [1]

- A kinetic energy
- B momentum
- C each body's velocity

2.3 For  $k=16$  and  $m=4$ , angular frequency is \_\_\_\_\_ rad/s. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: Zero torque about a point implies constant angular momentum \_\_\_\_\_ about it. [2]

### 3 Exam-style questions

**3.1** A bead of mass  $m$  moves without friction along a straight rail. Its potential is  $V(x) = ax^4 - bx^2$ , with  $a, b$  positive. Find every equilibrium, classify stability, and derive the small-oscillation frequency at each stable equilibrium. State why the frequency formula cannot describe motion near  $x = 0$ . [6]

## Solutions

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**2.1 B**  $\omega = \sqrt{k/m}$ .

Compare the rejected options: 2; 1/2. Use the stated model and conditions.

**2.2 B** Total momentum is conserved; kinetic energy is generally reduced.

Compare the rejected options: kinetic energy; each body's velocity. Use the stated model and conditions.

**2.3 2.**  $\omega = \sqrt{16/4} = 2$ .

**2.4** True.  $dL/dt$  equals the net torque about the fixed point.

**3.1** Force is  $F = -V' = -4ax^3 + 2bx$ . Equilibrium requires  $x = 0$  or  $x^2 = b/(2a)$ . Curvature is  $V'' = 12ax^2 - 2b$ : it is  $-2b$  at zero and  $4b$  at either outer point. Zero is unstable; both outer points are stable. Write  $x = x_0 + \xi$  and retain terms linear in  $\xi$ :  $m\ddot{\xi} = -4b\xi$ , so  $\omega = \sqrt{4b/m}$ . At zero the displacement grows rather than oscillates. The approximation requires  $|\xi|$  small compared with  $\sqrt{b/(2a)}$ . Award equilibrium, curvature, classification, linearisation, frequency and condition: one mark each.