1.1 Classical mechanics

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

KEY WORDS Lagrangian • angular momentum

Objective

Apply Newtonian, Lagrangian and Hamiltonian descriptions; Analyse rotation, central forces and oscillations; Use conservation laws with their conditions. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Newton's second law is a vector statement. Choose coordinates and identify constraints before components.

Energy conservation applies when work from nonconservative forces is accounted for; momentum conservation needs zero net external impulse.

For small oscillations, linearise around a stable equilibrium. A restoring term proportional to displacement gives harmonic motion.

Lagrange's equations use L=T−V and generalised coordinates. This undergraduate formalism is distinct from school-level force substitution.

■ Worked method

Choose an inertial frame and a coordinate x measured from spring equilibrium. For a conservative spring, the Lagrangian avoids solving constraint forces.

$$L = \tfrac{1}{2}m\dot{x}^2 - \tfrac{1}{2}kx^2.$$

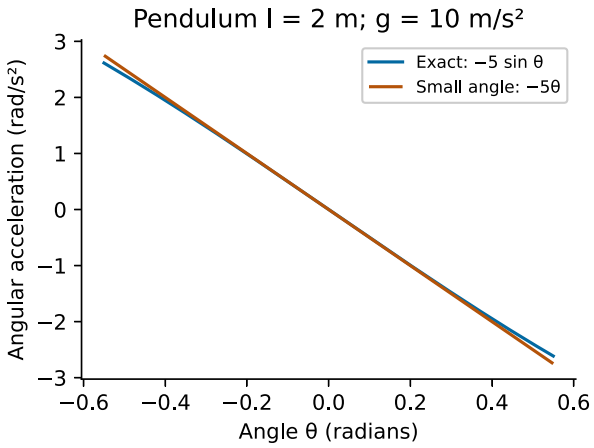
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0, \qquad m\ddot{x} + kx = 0.$$

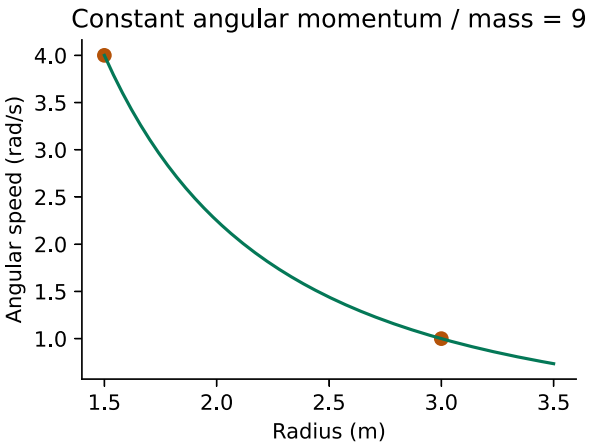
With m = 2.0 kg and k = 18 N/m,

$$\omega = \sqrt{k/m} = \sqrt{(18\,\mathrm{N/m})/(2.0\,\mathrm{kg})} = 3.0\,\mathrm{rad/s}.$$

$$T = 2\pi/\omega = 2\pi/(3.0\,\mathrm{rad/s}) = 2.09\,\mathrm{s}.$$

Positive stiffness gives stable motion. The units of k/m are inverse seconds squared.





2 Practice

2.1 Choose one answer. Doubling spring mass changes angular frequency by: [1]

- **A** 2
- **B** 1/sqrt(2)
- **C** 1/2

2.2 Choose one answer. In an isolated perfectly inelastic collision, conserved quantity is: [1]

- **A** kinetic energy
- **B** momentum
- **C** each body's velocity

2.3 For k=16 and m=4, angular frequency is _____ rad/s. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: Zero torque about a point implies constant angular momentum about it. [2]

3 Exam-style questions

3.1 A bead of mass m moves without friction along a straight rail. Its potential is $V(x) = ax^4 - bx^2$, with a,b positive. Find every equilibrium, classify stability, and derive the small-oscillation frequency at each stable equilibrium. State why the frequency formula cannot describe motion near x = 0. [6]

Solutions

2.1 B ω=sqrt(k/m).

Compare the rejected options: 2; 1/2. Use the stated model and conditions.

2.2 B Total momentum is conserved; kinetic energy is generally reduced.

Compare the rejected options: kinetic energy; each body's velocity. Use the stated model and conditions.

2.3 2. ω=sqrt(16/4)=2.

2.4 True. dL/dt equals the net torque about the fixed point.

3.1 Force is $F = -V' = -4ax^3 + 2bx$. Equilibrium requires $x = 0$ or $x^2 = b/(2a)$. Curvature is $V'' = 12ax^2 - 2b$: it is $-2b$ at zero and $4b$ at either outer point. Zero is unstable; both outer points are stable. Write $x = x_0 + \xi$ and retain terms linear in ξ: $m\ddot{\xi} = -4b\xi$, so $\omega = \sqrt{4b/m}$. At zero the displacement grows rather than oscillates. The approximation requires $|\xi|$ small compared with $\sqrt{b/(2a)}$. Award equilibrium, curvature, classification, linearisation, frequency and condition: one mark each.