

Exercise walk-through

31.1

Polarisation, analyser chains and phase

GRE · GRE Subject Test · GRE Physics

31.1 Polarisation, analyser chains and phase

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS Malus's law

- Brewster angle

Objective

Calculate successive ideal analyser transmissions from the immediate input state; Distinguish linear, circular and elliptical field motion using relative phase; Use Brewster incidence with refractive-index and plane-of-incidence conventions. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

An ideal analyser projects E onto its axis: amplitude $E \cos\theta$, intensity $I \cos^2\theta$. An ideal first filter passes $I_0/2$ for unpolarised input and makes the output linear.

Update polarisation after every filter. Unpolarised input through axes 0° , 45° , 90° gives

$I_0/2$, $I_0/4$, $I_0/8$. A middle filter can restore transmission through an otherwise crossed pair.

Equal-amplitude transverse components in quadrature give circular polarisation. Same/opposite phase gives linear; unequal quadrature components give elliptical. A quarter-wave plate needs the appropriate input direction.

Brewster: $\tan\theta_B = n_2/n_1$ from the normal. Reflected p vanishes; reflected s survives. Use Snell's law for refraction; do not substitute the total-internal-reflection critical-angle rule.

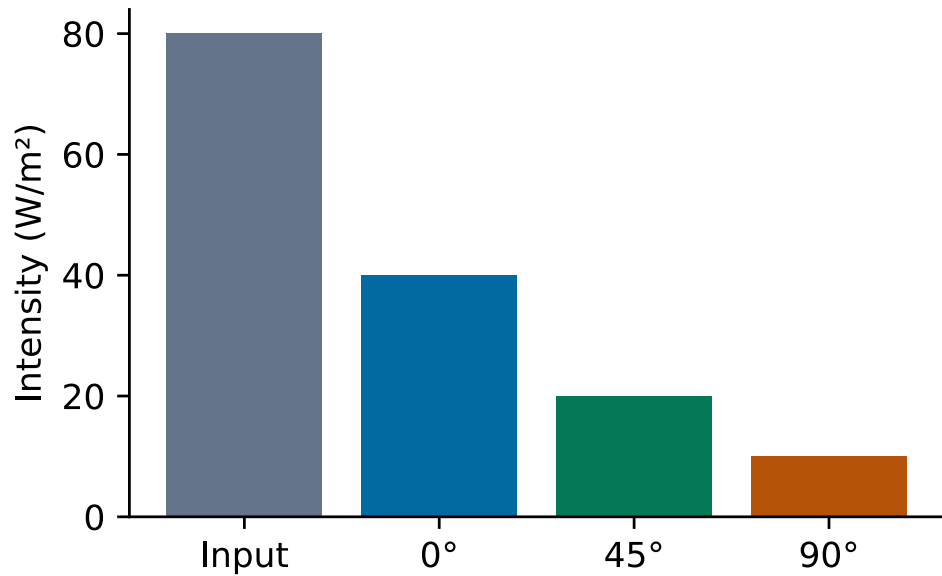
■ Worked method

An ideal first polariser transmits half of unpolarised incident intensity I_0 . Each later analyser uses the angle from the immediately preceding axis. For axes 0, 30 and 90 degrees,

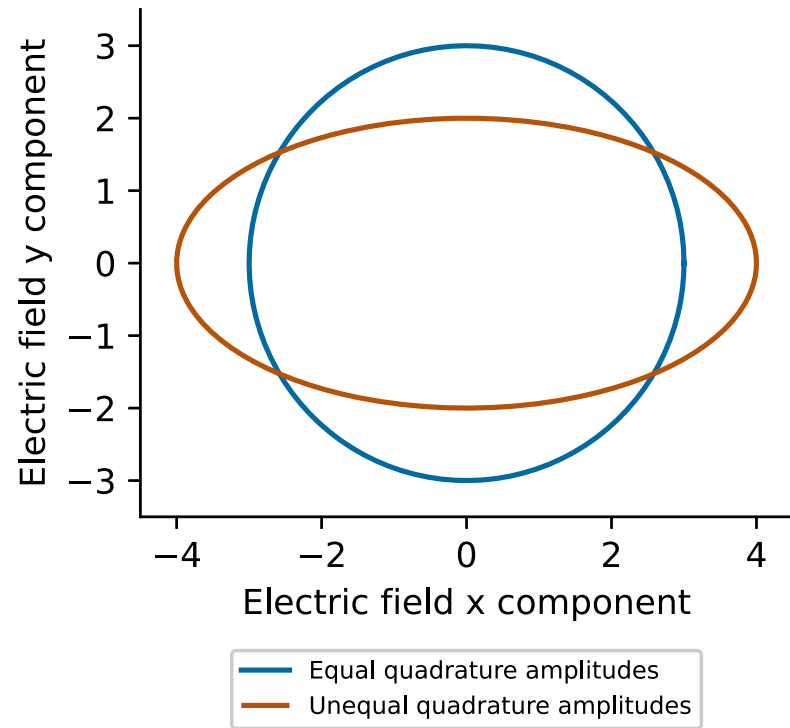
$$I_f = \frac{1}{2}I_0 \cos^2(30^\circ) \cos^2(60^\circ) = 3I_0/32.$$

The field after each filter points along that filter's axis. Using only the first and last axes would predict zero incorrectly.

Unpolarised input; ideal analyser chain



Circular and elliptical field paths



Exercise 2.1

2.1 Choose one answer. Unpolarised intensity I_0 passes ideal axes 0° , 30° , 90° . Final intensity is: [1]

- **A** 0
- **B** $I_0/8$
- **C** $3I_0/32$
- **D** $3I_0/8$
- **E** $I_0/2$

Exercise 2.1

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- **C** $3I_0/32$
- **D** $3I_0/8$
- **E** $I_0/2$

2.1 C The first output is $I_0/2$; later angles are 30° and 60° . Multiply $(1/2)(3/4)(1/4)=3/32$.

Compare the rejected options: 0; $I_0/8$; $3I_0/8$; $I_0/2$. Use the stated model and conditions.

Exercises 2.2·2.3

2.2 Choose one answer. At one point $E_x = 2 \cos \omega t$ and $E_y = 2 \sin \omega t$. The polarisation is: [1]

- **A** Linear along x
- **B** Linear at 45°
- **C** Circular
- **D** Unpolarised
- **E** Longitudinal

2.3 Polarised input intensity 24 W/m^2 meets an ideal analyser at 60° . Output is _____ W/m^2 . Show your relation and units. [2]

Exercises 2.2·2.3

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- **A** Linear along x
- **B** Linear at 45°
- **C** Circular
- **D** Unpolarised
- **E** Longitudinal

2.2 C Equal nonzero amplitudes with quarter-cycle phase give constant magnitude and a rotating transverse direction.

Compare the rejected options: Linear along x; Linear at 45° ; Unpolarised; Longitudinal. Use the stated model and conditions.

2.3 Polarised input intensity 24 W/m^2 meets an ideal analyser at 60° . Output is _____ W/m^2 . Show your relation and units. [2]

2.3 6. $24 \cos^2 60^\circ = 24/4 = 6$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: At Brewster incidence for the ideal dielectric model, reflected light from unpolarised input is polarised perpendicular to the plane of incidence. [2]

3.1 Insert one ideal analyser at angle θ between two crossed ideal polarisers. Derive final intensity for unpolarised input I_0 and find the angle giving maximum output. Compare with removing the middle analyser. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: At Brewster incidence for the ideal dielectric model, reflected light from unpolarised input is polarised perpendicular to the plane of incidence. [2]

2.4 True. The reflected p component is zero; the surviving s electric field is perpendicular to the incidence plane.

3.1 Insert one ideal analyser at angle θ between two crossed ideal polarisers. Derive final intensity for unpolarised input I_0 and find the angle giving maximum output. Compare with removing the middle analyser. [4]

3.1 $I_f = (I_0/2) \cos^2 \theta \sin^2 \theta = (I_0/8) \sin^2(2\theta)$. The maximum is $I_0/8$ at 45 degrees between the outer axes (modulo equivalent axes). Removing the middle analyser gives zero. It changes polarisation while reducing intensity; it does not create energy. Award chain, identity, optimum and comparison/interpretation.