

Exercise walk-through

# 19.1

## Gas processes, entropy and reversible cycles

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GRE · GRE Subject Test · GRE Physics

# 19.1 Gas processes, entropy and reversible cycles

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 11 marks**

**KEY WORDS** entropy production • coefficient of performance

# Objective

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Apply first-law work and heat signs to gas processes; Compare reversible entropy balances and P–V cycle areas; Distinguish engine efficiency from refrigerator coefficient of performance . Original public practice; not an ETS paper.

## 1 Key knowledge and worked examples

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Use  $\Delta U = Q - W$ , with heat entering and work done by the gas positive. Quasistatic work is  $\int P \, dV$ . Ideal-gas  $U$  depends on temperature: constant volume gives  $Q = \Delta U$ ; isothermal gives  $Q = W$ ; adiabatic gives  $Q = 0$ . A complete cycle has  $\Delta U = 0$ , hence net  $Q = \text{net } W$ .

Reversible isothermal paths have  $PV$  constant; with constant heat capacities, adiabatic paths have  $PV^\gamma$  constant and fall more steeply. Isobaric paths are horizontal. Clockwise

P–V cycles give positive work: rectangle area  $\Delta P \Delta V$ , triangle area half that. Convert litres to  $\text{m}^3$ .

Reversible heat transfer gives  $dS = \delta Q_{\text{rev}}/T$ . Total system-plus-environment entropy change is zero, but system entropy can change: isothermal expansion gives  $nR \ln(V_2/V_1)$ , balanced by reservoir entropy loss. Irreversibility produces nonnegative total entropy; reversible does not mean isothermal or zero work.

Carnot engine efficiency is  $1 - T_C/T_H$ . Refrigerator  $\text{COP} = Q_C/W = T_C/(T_H - T_C)$ ; heat-pump  $\text{COP} = T_H/(T_H - T_C)$ , using kelvin. On a reversible T–S rectangle, isothermal heat is  $T \Delta S$  and work magnitude is the area. Refrigerator COP can exceed one because it moves heat while consuming work.

## ■ Worked method

A reversible refrigerator transfers entropy  $\Delta S$  from  $T_c$  to  $T_h$ .

$$Q_c = T_c \Delta S = (270 \text{ K})(3.0 \text{ J/K}) = 810 \text{ J}.$$

$$Q_h = T_h \Delta S = (300 \text{ K})(3.0 \text{ J/K}) = 900 \text{ J}.$$

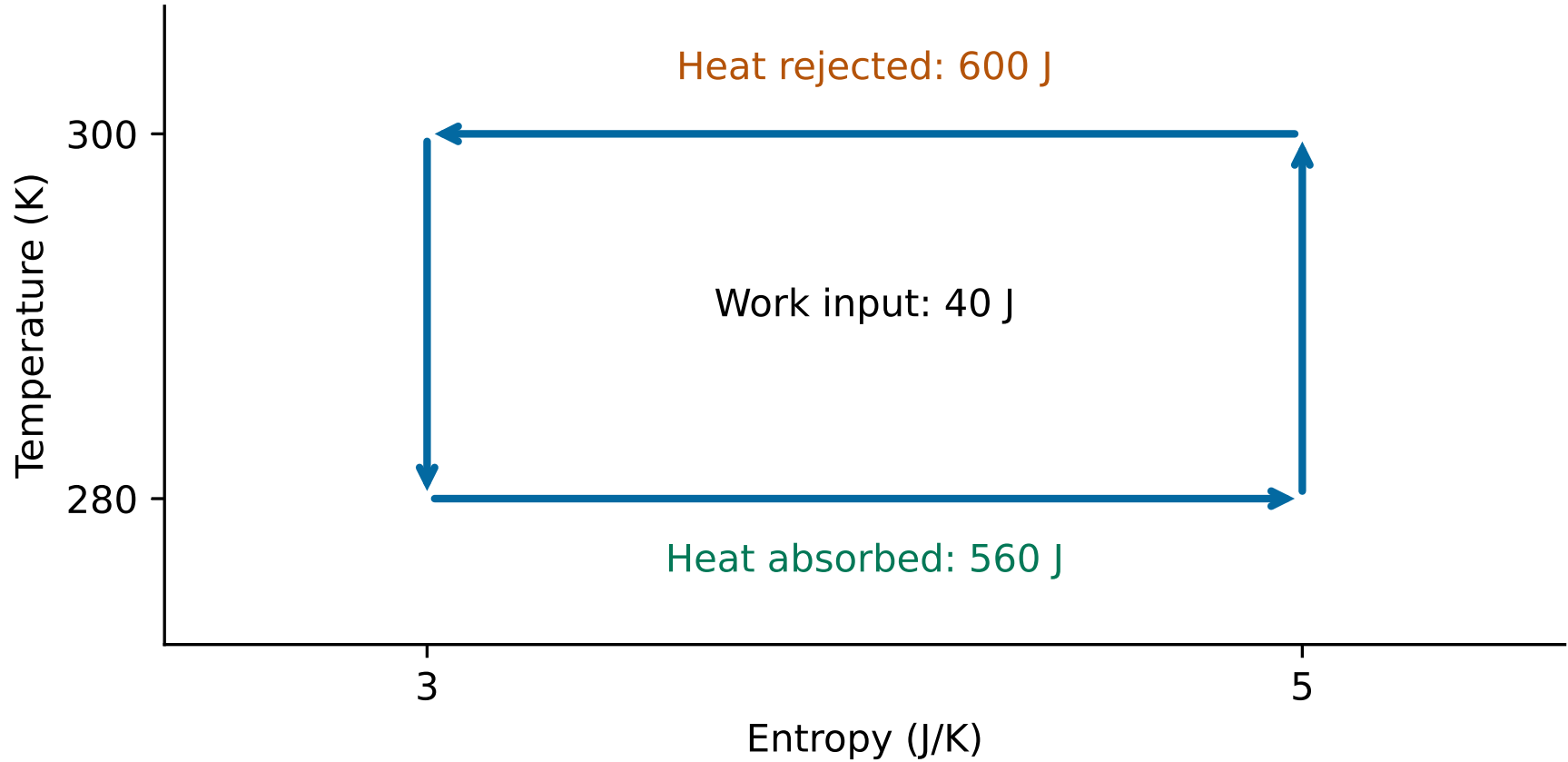
$$W_{in} = Q_h - Q_c = 900 \text{ J} - 810 \text{ J} = 90 \text{ J}.$$

$$\text{COP} = Q_c / W_{in} = (810 \text{ J}) / (90 \text{ J}) = 9.$$

COP above one is possible because heat is moved, rather than all produced from work.

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Reversible refrigerator:  $\Delta S = 2 \text{ J/K}$ ; COP = 14



## Exercise 2.1

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**2.1** Choose one answer. In a reversible process of a system and its environment, which must hold? [1]

- **A** The system entropy is constant
- **B** Temperature is constant
- **C** Total entropy change is zero
- **D** Work is zero
- **E** Internal energy is constant

## Exercise 2.1

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**2.1** Choose one answer. In a reversible process of a system and its environment, which must hold? [1]

- **A** The system entropy is constant
- **B** Temperature is constant
- **C** Total entropy change is zero
- **D** Work is zero
- **E** Internal energy is constant

**2.1 C** Reversibility produces no total entropy; individual system quantities may change.

Compare the rejected options: The system entropy is constant; Temperature is constant; Work is zero; Internal energy is constant. Use the stated model and conditions.

## Exercises 2.2·2.3

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**2.2** Choose one answer. A reversible refrigerator has  $T_C=270$  K and  $T_H=300$  K. Its COP is: [1]

- A 0.1
- B 0.9
- C 1
- D 9
- E 10

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**2.3** A clockwise rectangular P–V cycle has  $\Delta P=4000$  Pa and  $\Delta V=0.002$  m<sup>3</sup>. Net work is \_\_\_\_\_ J. Show your relation and units. [2]

## Exercises 2.2·2.3

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**2.2** Choose one answer. A reversible refrigerator has  $T_C=270$  K and  $T_H=300$  K. Its COP is: [1]

- A 0.1
- B 0.9
- C 1
- D 9
- E 10

**2.2 D**  $Q_C/W=270/(300-270)=9$ .

Compare the rejected options: 0.1; 0.9; 1; 10. Use the stated model and conditions.

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**2.3** A clockwise rectangular P–V cycle has  $\Delta P=4000$  Pa and  $\Delta V=0.002$  m<sup>3</sup>. Net work is \_\_\_\_\_ J. Show your relation and units. [2]

**2.3 8.** Positive clockwise work is  $\Delta P\Delta V=8$  J.

## Exercises 2.4·3.1

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**2.4** Decide whether this claim is true. Justify it: For any complete gas cycle, net entering heat equals net work done by the gas. [2]

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**3.1** A refrigerator removes 600 J from 280 K and rejects 650 J to 300 K per cycle. Check energy balance and total entropy change. Is the proposed cycle possible? Find the minimum work needed for the same cooling between these reservoirs. [5]

## Exercises 2.4·3.1

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**2.4** Decide whether this claim is true. Justify it: For any complete gas cycle, net entering heat equals net work done by the gas. [2]

**2.4** True.  $\Delta U=0$  because internal energy is a state function and the initial state returns.

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**3.1** A refrigerator removes 600 J from 280 K and rejects 650 J to 300 K per cycle. Check energy balance and total entropy change. Is the proposed cycle possible? Find the minimum work needed for the same cooling between these reservoirs. [5]

**3.1** Energy requires 50 J work input. Reservoir entropy change is  $650/300 - 600/280 = +0.02381 \text{ J/K} > 0$ , so the complete cycle does not violate the second law. Reversible minimum work is  $Q_c(T_h/T_c - 1) = 600(300/280 - 1) = 42.86 \text{ J}$ . The proposed cycle is possible thermodynamically and uses more than minimum work. Award work, correct entropy arithmetic, sign/possibility, reversible bound and comparison.