Course notes

GRE Subject Test · GRE Physics

Gre Physics

Computer-delivered undergraduate GRE Physics

Approximately 70 multiple-choice items in 120 minutes. Mechanics, electromagnetism, optics/waves, thermal/statistical physics, quantum/atomic physics, relativity, laboratory methods and specialised topics. No A-level equivalence is claimed.

Official scope: https://www.ets.org/gre/test-takers/subject-tests/about/content-structure.html

CM · Classical mechanics

- Newton's second law is a vector statement. Choose coordinates and identify constraints before components.
- Energy conservation applies when work from nonconservative forces is accounted for; momentum conservation needs zero net external impulse.
- For small oscillations, linearise around a stable equilibrium. A restoring term proportional to displacement gives harmonic motion.
- Lagrange's equations use L=T−V and generalised coordinates. This undergraduate formalism is distinct from school-level force substitution.

■ Checked example

For a mass m on a spring k, L=(1/2)m xdot^2−(1/2)kx^2. Euler–Lagrange gives m xddot+kx=0. Thus angular frequency is sqrt(k/m). Doubling mass reduces frequency by factor sqrt(2), not two.

■ Common error

Conservation of kinetic energy does not hold in every collision, even when momentum is conserved.

EM · Electromagnetism

- Electric field is the negative gradient of potential. A value and a spatial change are different quantities.
- Use Gauss's law with symmetry. A known total flux alone does not give the field at every surface point.
- Kirchhoff's laws track charge and energy in circuits. Induced emf follows changing magnetic flux with the Lenz-law sign.
- Maxwell equations link electric and magnetic fields. In vacuum a wave has E/B=c and carries energy.

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■ Checked example

Inside a spherical shell with uniformly distributed charge, symmetry and Gauss's law give E=0. Its potential is constant inside, not necessarily zero. For an RC discharge, $V(t)=V_0 \exp(-t/RC)$, and after one time constant it is V_0/e.

■ Common error

Do not use Gauss's law to claim a uniform field on an arbitrary nonsymmetric surface.

WO · Waves and optics

- Add wave amplitudes, then calculate intensity. Intensity is proportional to squared amplitude for the same medium.
- A path difference gives phase difference $2\pi\Delta/\lambda$. Coherent sources need stable relative phase.
- For a single slit, minima satisfy $a \sin\theta = m\lambda$ for nonzero integers m. A double slit has a different interference condition.
- Geometric optics needs a stated sign convention. Wave effects matter when apertures are comparable to wavelength.

■ Checked example

Two equal coherent waves have amplitude A each. In phase, their total amplitude is 2A and intensity is $4I_0$. With phase difference π, amplitude and ideal intensity are zero. Adding intensities directly misses interference.

■ Common error

The central single-slit maximum is not obtained by setting m=0 in the minima condition.

TS · Thermodynamics and statistical mechanics

- State the first-law sign convention: $\Delta U = Q - W$ when W is work done by the system.
- For a reversible engine between reservoirs, efficiency is bounded by $1 - T_c/T_h$ using absolute temperatures.
- Boltzmann weights are $\exp(-E/kT)$. Probabilities require division by the partition function.
- Distinguish Maxwell–Boltzmann, Bose–Einstein and Fermi–Dirac assumptions. Fermions cannot share an identical single-particle state.

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■ Checked example

Reservoirs at 600 K and 300 K give a maximum reversible efficiency 1−300/600=0.5. An engine taking 1000 J can produce at most 500 J of work under these conditions. Conservation alone would not impose that bound.

■ Common error

Celsius temperature ratios cannot be used in the Carnot efficiency formula.

QM · Quantum mechanics and atomic physics

- A state wavefunction must satisfy the model's boundary conditions and have total probability one: integrate $|\psi(x)|^2$ over the allowed domain. For orthonormal states φ_j, a superposition $\psi=\Sigma c_j\varphi_j$ has norm squared $\Sigma|c_j|^2$ because the cross terms integrate to zero. Thus an equal superposition of N orthonormal states has coefficient magnitude 1/sqrt(N). The probability of finding one constituent energy is $|c_j|^2$, not c_j. Relative phases may affect position probabilities even when energy probabilities are unchanged.
- An observable is represented by a Hermitian operator. Its eigenvalues are real, and a normalised state gives expectation value $\langle A\rangle=\int\psi^*\,A\psi\,dx$. The expectation is an average over repeated preparations, not necessarily the result of one measurement. If A satisfies a polynomial identity, apply it to an eigenstate: for $A^4=I$, a real eigenvalue must be +1 or −1. A complex fourth root is excluded by Hermiticity even though it solves the polynomial.
- For an infinite well $0<x<L$, $\psi(0)=\psi(L)=0$ allows sine modes $\sin(n\pi x/L)$, with n=1,2,… . Their energies are $E_n=n^2\pi^2\hbar^2/(2mL^2)$, so the ground state is not zero and doubling L divides every energy by four. The equivalent h expression is $n^2h^2/(8mL^2)$; never substitute h for $\hbar$ without its 2π factor. A finite barrier instead permits exponential tails, and a travelling wave in a classically allowed region has oscillatory rather than exponentially decaying spatial dependence.
- Operators commute when [A,B]=AB−BA=0; compatible observables can have a common eigenbasis. Since kinetic energy $p^2/(2m)$ is a function of p, it commutes with p. A position-dependent potential generally makes $H=p^2/(2m)+V(x)$ fail to commute with p. Angular momentum obeys $[J_x,J_y]=i\hbar J_z$ and cyclic permutations, while J^2 commutes with each component. Atomic transitions exchange positive photon energy equal to a level difference; angular momentum and mechanism-specific selection rules restrict which transitions occur.

■ Checked example

For an infinite well, E2=4E1 and E3=9E1. A transition from n=3 to n=2 releases 5E1. Doubling L lowers every energy by factor four. n=0 would give the zero wavefunction, not a physical state.

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■ Common error

A stationary state can have a nonzero energy even though its probability density is time-independent.

RA · Relativity, laboratory methods and specialised topics

- Special relativity uses $\gamma=1/\mathrm{sqrt}(1-v^2/c^2)$. Rest energy is mc^2 and total energy is γmc^2.
- For independent small uncertainties, combine contributions in quadrature; correlated uncertainties require covariance terms.
- Random scatter affects precision; calibration bias affects accuracy. A log plot can reveal a power law.
- Nuclear, particle, condensed-matter and astrophysics questions require undergraduate vocabulary and models. School physics alone is not a complete preparation course.

■ Checked example

For $v=0.6c$, $\gamma=1/\mathrm{sqrt}(0.64)=1.25$. Total energy is $1.25mc^2$; kinetic energy is $(\gamma-1)mc^2=0.25mc^2$. For $y=ab$ with independent small relative uncertainties 3% and 4%, relative uncertainty is $\mathrm{sqrt}(3^2+4^2)\%=5\%$.

■ Common error

Adding percentage uncertainties linearly gives a worst-case bound, not the independent statistical standard uncertainty.

AT · Atomic spectra and selection rules

- For hydrogen-like one-electron ions, the bound energies scale as $-13.6\ Z^2/n^2$ eV in the simplest nonrelativistic model. Emission requires a downward transition and positive photon energy.
- Orbital angular momentum has $l=0,\ldots,n-1$ and $m_l=-l,\ldots,l$. Electron spin adds $s=1/2$; total angular momentum combines orbital and spin contributions.
- Electric-dipole selection rules include $\Delta l=\pm1$ and $\Delta m=0,\pm1$. These rules concern a specific transition mechanism; forbidden does not mean impossible by every mechanism.
- Fine structure, spin–orbit effects and Zeeman splitting refine the simplest hydrogen picture. In many-electron atoms, shielding and electron interactions invalidate direct hydrogen-energy substitution.

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■ Checked example

In hydrogen, E1=−13.6 eV and E2=−3.4 eV. A 2p→1s emission releases 10.2 eV and satisfies Δl=−1. A 2s→1s transition has Δl=0, so it is electric-dipole forbidden even though the same energy difference exists.

■ Common error

Do not make the final-state energy minus initial-state energy negative for an emitted photon.

SN · Solid-state, nuclear and particle models

- Filled bands cannot carry ordinary current without accessible nearby states. Metals have partially filled or overlapping bands; semiconductors have a small band gap. Donor and acceptor doping change carrier populations.
- Fermi–Dirac occupancy is 1/[exp((E−μ)/(kT))+1]. At zero temperature it becomes a step at the Fermi energy; thermal broadening does not make every state half occupied.
- Radioactive populations obey N=N0e^(−λt), activity λN and half-life ln2/λ. Binding energy comes from a mass deficit multiplied by c^2; compare binding energy per nucleon when discussing stability.
- Nuclear and particle processes must conserve charge, energy, momentum and the applicable quantum numbers. Beta decay includes a neutrino or antineutrino; a two-body electron-only story cannot explain its continuous energy spectrum.

■ Checked example

After three half-lives, N/N0=(1/2)^3=1/8, so activity is also one eighth if λ is unchanged. A mass defect of 0.010 u corresponds to about 9.315 MeV using 1 u c^2=931.5 MeV.

■ Common error

Activity and surviving population both decay, but their units differ: activity is transitions per second.

AH · Hamiltonian mechanics and astrophysical scaling

- The canonical momentum is p=∂L/∂qdot. The Legendre transform H=p qdot−L gives a Hamiltonian when velocities can be expressed using coordinates and momenta.
- Hamilton's equations are qdot=∂H/∂p and pdot=−∂H/∂q. For an ordinary oscillator H=p^2/(2m)+kq^2/2, these reproduce Newton's equation.
- For a circular gravitational orbit v^2=GM/r and period^2=4$\pi^2 r^3$/(GM). The assumptions include a dominant central mass and a circular approximation; elliptical orbits

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use the semimajor axis in Kepler's law. For a circular satellite of mass m, angular momentum magnitude is mrv=m sqrt(GMr), so identical satellites have L proportional to sqrt(r). A radius ratio 9 therefore gives angular-momentum ratio 3, while the period ratio is 27. This distinction follows from the same centripetal-force relation; do not use period scaling for angular momentum.

- Luminosity and received flux obey $F=L/(4\pi d^2)$ for isotropic radiation. A blackbody has $L=4\pi R^2\sigma T^4$; its spectral peak shifts inversely with temperature. Distinguish intrinsic luminosity from observed brightness.

■ Checked example

For the oscillator, qdot=p/m and pdot=−kq, hence qddot=−(k/m)q. If an otherwise identical star is twice as far away, received flux is one quarter, although luminosity is unchanged.

■ Common error

A Hamiltonian equals total mechanical energy only under the relevant system assumptions; do not infer this universally from its name.

CM.1 · Collisions, work and oscillator energy

- Choose an isolated system during the short collision. With negligible external impulse, conserve total momentum separately in every Cartesian direction. If masses m1 and m2 stick, their common velocity is (m1 v1+m2 v2)/(m1+m2). Find the magnitude only after adding vectors. Initial kinetic energy is the sum of each $\frac{1}{2}m|v|^2$; final kinetic energy uses the total mass and the common speed. Their difference becomes internal energy or deformation, not missing momentum. A perfectly inelastic collision means sticking, not final rest.
- The work–energy theorem $\Delta K=\int F\cdot dr$ uses net force along displacement. For a constant parallel force over distance s, ΔK=Fs. Speed doubling quadruples kinetic energy. Conservation of mechanical energy is a separate statement requiring no unaccounted nonconservative work. At a spring displacement x, $U=\frac{1}{2}kx^2$. Under the same applied force F, equilibrium extension is F/k and stored energy is $F^2/(2k)$: a stiffer spring stretches less and stores less energy. Under the same extension, it stores more. Identify which condition is held fixed.
- Near a stable equilibrium x0, expand $U\approx U(x0)+\frac{1}{2}U''(x0)(x-x0)^2$; the effective stiffness is positive U″(x0). The motion obeys m xddot=−k_eff(x−x0), with ω=sqrt(k_eff/m) and period $2\pi/\omega$. A simple pendulum obeys θddot+(g/L)sinθ=0. For small angles $\sin\theta\approx\theta$, so $T=2\pi\sqrt{L/g}$. This approximation explains period–length scaling; a large amplitude requires a correction, and the mass cancels for an ideal pendulum.
- For an undamped harmonic oscillator, total energy is $\frac{1}{2}mv^2+\frac{1}{2}kx^2=\frac{1}{2}kA^2$. At equilibrium all its energy is kinetic; at a turning point its speed is zero and all its

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energy is potential. Angular frequency is not ordinary frequency: $\omega=2\pi f$. If the same oscillator has equilibrium speed vmax, A=vmax/ω. Distinguish the instant at which speed is measured from the release displacement, and keep SI units when a millijoule answer is requested.

Checked example

A 2 kg cart at (3,0) m/s sticks to a 1 kg cart at (0,3) m/s. Momentum is (6,3) kg · m/s, so final velocity is (2,1) m/s. Initial K=9+4.5=13.5 J; final K=½ · 3 · 5=7.5 J. Internal-energy increase is 6 J. Separately, a 0.5 kg oscillator has equilibrium speed 0.2 m/s: its total energy is 0.01 J, or 10 mJ.

Common error

Do not add incoming speed magnitudes as momenta, or conserve kinetic energy in a sticking collision. Fixed-force and fixed-extension spring comparisons have opposite answers.

CM.2 · Rotation, buoyancy and terminal-speed balances

- Torque about a fixed pivot is r×F; its magnitude uses the perpendicular lever arm. For a uniform rod of length L pivoted at one end, gravity acts at its centre L/2 and the moment of inertia is $ML^2/3$. If its angle θ is measured from vertical, gravitational torque magnitude is $Mg(L/2)\sin\theta$, so angular-acceleration magnitude is $3g\sin\theta/(2L)$. Define the positive rotation direction before assigning a sign. Using the centre-of-mass inertia $ML^2/12$ without the parallel-axis correction gives a wrong acceleration.
- An object rolling without slipping satisfies v=Rω. Write total kinetic energy as $\frac{1}{2}Mv^2+\frac{1}{2}I\omega^2$. With I=$\beta MR^2$, the rotational fraction is $\beta/(1+\beta)$; for a solid disk $\beta=1/2$ and the fraction is 1/3. A hoop has $\beta=1$ and fraction 1/2. Static friction can supply the torque required for rolling without dissipating energy at an instantaneously stationary contact on a fixed surface. A sliding object does not satisfy the no-slip relation automatically.
- For a floating composite, buoyancy equals the weight of displaced fluid, summed over every submerged part. Let a wood block have volume V and a dense stone volume Vs. If the stone is above the water, only submerged wood contributes: ρwater g fV equals total weight. If the stone is attached below and fully submerged, its displaced volume reduces the wood's required submerged fraction to f−Vs/V. The stone does not cease to weigh anything; the water now supplies some of its support. Check whether the assumed orientation and flotation are physically possible.
- Terminal speed means zero acceleration, not zero speed or zero force. With quadratic drag C v^2 and a downward weight Mg, neglecting buoyancy gives vt=sqrt(Mg/C). Two otherwise identical balls share the drag coefficient C; doubling mass then multiplies terminal speed by sqrt(2). If buoyancy matters, replace Mg by (M−ρfluid V)g. A linear-drag model instead gives a different mass

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scaling. State the specified drag law and compare force balances, rather than transferring a formula between models.

Checked example

A solid disk rolls without slipping: $I=MR^2/2$ gives rotational $K=\frac{1}{4}Mv^2$, translational $K=\frac{1}{2}Mv^2$ and rotational fraction 1/3. A wood–stone composite initially displaces 0.60V through wood alone. If the stone has volume 0.10V and is then fully underwater, the required submerged wood volume is 0.50V. Total displaced volume remains 0.60V.

Common error

Use the inertia about the chosen pivot. In flotation, include the stone's displaced volume as well as its weight. Terminal speed requires a specified drag law.

CM.3 · Coupled acceleration and relative motion

- For a mass m1 on a frictionless horizontal table connected over an ideal massless pulley to a hanging mass m2, choose table motion and downward hanging motion as positive. A taut inextensible string gives one acceleration magnitude. Write T=m1a and m2g−T=m2a. Adding eliminates the internal tension, so a=m2g/(m1+m2) and T=m1m2g/(m1+m2). Thus T<m2g when both masses are positive. Setting tension equal to hanging weight silently assumes zero acceleration.
- The common tension assumption requires an ideal massless string and pulley with negligible friction and inertia. A pulley with rotational inertia can have different tensions, with $(T2-T1)R=I\alpha$ and $a=R\alpha$ if the string does not slip. Draw each body separately: a force internal to the combined system remains an external force on one chosen body. Constraint forces can cancel from a system equation while still being needed for individual motion.
- For two inertial frames with constant relative velocity V, Galilean velocity transformation is v_relative=v_ground−V. A payload released by a level aircraft initially has the aircraft's horizontal speed. With negligible air resistance, horizontal ground velocity remains constant and vertical velocity becomes −gt if upward is positive. Relative to that aircraft, horizontal velocity is zero and downward speed is gt. Position is a different question: vertical displacement is $-\frac{1}{2}gt^2$. Do not confuse ground speed with relative speed.
- Check limiting cases after solving. As m1 approaches zero, the ideal coupled acceleration approaches g and tension approaches zero; as m1 grows very large, acceleration approaches zero and tension approaches m2g from below. An energy derivation gives $m2g\ s=\frac{1}{2}(m1+m2)v^2$ for release from rest and an ideal pulley, agreeing with $v^2=2as$. This agreement checks the equations; it does not permit energy conservation if friction or other unaccounted work is present.

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■ Checked example

With m1=3 kg, m2=2 kg and g=10 m/s^2, acceleration is 4 m/s^2 and tension is 12 N, below the hanging weight 20 N. From rest after a 0.5 m descent, $v^2=2\cdot 4\cdot 0.5=4$, so v=2 m/s. A payload released by an aircraft at constant horizontal speed 80 m/s has ground velocity (80,−30) m/s after 3 s, but relative velocity (0,−30) m/s.

■ Common error

A massless ideal pulley equates tension, not tension and weight. Subtract frame velocities component by component before taking a speed magnitude.

EM.1 · Electrostatic superposition, flux and conductors

- For a point charge Q at position r0, $E(r)=Q(r-r0)/(4\pi\varepsilon 0|r-r0|^3)$. Superpose vectors, not field magnitudes. For two positive charges attracting an electron, forces in the same direction add; opposite directions subtract; perpendicular components combine by Pythagoras. Define the observation point and the direction of each force first. Potential is the scalar sum $V=\Sigma Q/(4\pi\varepsilon 0r)$ with zero at infinity. A zero potential does not generally mean a zero field, since $E=-\nabla V$.
- Gauss's law is the closed-surface integral $\oint E\cdot dA=Q_enclosed/\varepsilon 0$. It determines a local field simply only when symmetry makes the normal field uniform or makes other flux contributions zero. A charge near an infinite plane sends half its total flux through that plane in magnitude: the plane subtends solid angle 2π out of 4π. The result is independent of distance and lateral position, but its sign depends on the chosen plane normal and charge sign. An open plane does not enclose a charge; using closed-surface wording for it is incorrect.
- In electrostatic equilibrium, the electric field inside conducting material is zero and the conductor has constant potential. Place a charge Q at the centre of a conducting spherical shell with inner radius a, outer radius b and shell net charge q. A Gaussian surface inside the material requires inner-surface charge −Q, so the outer surface has q+Q. Spherical symmetry makes the external field that of total Q+q at the centre. The material's potential, zero at infinity, is $(Q+q)/(4\pi\varepsilon 0b)$, independent of the material observation radius.
- In the cavity of that centred-charge shell, the field is $Q/(4\pi\varepsilon 0r^2)$ radially for $0<r<a$. Potential includes both the point-charge contribution and constant shell contributions; continuity holds across each surface even though the normal field jumps at a surface charge. Keep cavity, conductor material and exterior separate. An off-centre charge still induces total inner charge −Q, but its cavity field is no longer the simple centred radial field. Zero interior conductor field follows equilibrium, not an assumption that every surface charge distribution is uniform.

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■ Checked example

Take Q=+2 nC, shell net charge q=−1 nC and b=0.30 m. The inner surface has −2 nC; the outer surface has +1 nC. Using $1/(4\pi\varepsilon 0)=9\times10^9$, the potential everywhere in the conducting material is 30 V, while its field is zero. Separately, attraction components 3 N right and 4 N up combine into magnitude 5 N, not 7 N.

■ Common error

Do not confuse the open-plane half-flux result with enclosed charge. A conductor's zero field fixes a constant potential, not necessarily zero potential.

EM.2 · Circuit power, induction and charged-particle motion

- For steady ideal resistor circuits, series resistances add and parallel conductances add. Reduce a network while preserving which nodes share the same voltage. With a 24 V source, a 2 Ω series resistor and parallel 3 Ω and 6 Ω branches, the parallel equivalent is 2 Ω. Total current is 6 A, branch voltage is 12 V and branch currents are 4 A and 2 A. The current through every component is not necessarily the total current; verify Kirchhoff current and voltage sums after reduction.
- For sinusoidal current I=Imax sin(ωt−φ), Irms=Imax/sqrt(2). A series RLC circuit has impedance magnitude sqrt($R^2+(\omega L-1/(\omega C))^2$) and average dissipated power $I_{rms}^2R=V_{rms}I_{rms}\cos\varphi$. Ideal inductors/capacitors exchange stored energy with zero average dissipation. If rms current is already given, do not insert another factor 1/2. An ideal rectifying diode conducts in its forward direction and blocks in reverse: a suitably oriented series diode clips the negative half of a sinusoidal resistor voltage; it does not create a full-wave rectifier by itself.
- Magnetic flux is $\int B\cdot dA$, or $BA\cos\theta$ for a uniform field with θ measured from the area normal. Faraday emf is $-N\,d\Phi/dt$. For changing perpendicular field and fixed loop geometry, its average magnitude is $NA|\Delta B|/\Delta t$. Determine the sign or current direction using opposition to the flux change, not opposition to the field itself. The magnetic field at the centre of one circular loop is $\mu 0I/(2R)$, obtained from Biot–Savart: every element contributes in the same axial direction. An N-turn compact coil multiplies this result by N.
- For a nonrelativistic charge with velocity perpendicular to uniform B, magnetic force $|q|vB$ provides centripetal force mv^2/r. Thus $r=mv/(|q|B)$ and cyclotron angular frequency is $|q|B/m$, independent of speed in this approximation. Charge sign sets rotation direction. A deuteron has approximately twice proton mass and the same charge, while an alpha particle has four times its mass and twice its charge; their frequencies are each half the proton value. In crossed E and B fields, undeflected motion requires electric and magnetic forces opposite and v=E/B. Check vector directions before using this magnitude.

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■ Checked example

In the 24 V network, 2 Ω in series with (3 Ω parallel 6 Ω) gives total 4 Ω, current 6 A and the 6 Ω branch current 2 A. A 100-turn coil of area 0.020 m^2 has perpendicular field rise by 0.030 T in 0.10 s: average emf magnitude is 0.60 V. A velocity selector with E=6000 V/m and B=0.20 T selects speed 30000 m/s.

■ Common error

Do not apply battery voltage to every branch or halve an rms-current power again. Flux angle uses the area normal; a selector also needs opposing force directions.

EM.3 · Maxwell waves, energy flow and boundary conditions

- Ampere–Maxwell law in vacuum is $\oint B \cdot dl = \mu 0(I_conduction + \varepsilon 0\ d\Phi E/dt)$. The added displacement-current term depends on the time rate of electric flux through the chosen surface. It ensures consistent results for a loop whose spanning surface either crosses a capacitor wire or passes between the plates. Displacement current is not magnetic flux or an integral over earlier electric flux. For a uniform plate field, the gap contribution is $\varepsilon 0 A\ dE/dt$; signs follow the oriented surface.
- In a homogeneous, linear, lossless dielectric with permeability μ and permittivity ε, Maxwell equations give wave speed $v = 1/\mathrm{sqrt}(\mu\varepsilon)$ and refractive index $n = c/v = \mathrm{sqrt}(\mu r \varepsilon r)$. For nonmagnetic material $\mu r \approx 1$, so $v = c/\mathrm{sqrt}(\varepsilon r)$. Use the stated frequency-dependent material constants when dispersion matters; a static dielectric constant need not describe every optical frequency. Vacuum waves have E/B=c; in this simple medium the relation is E/B=v.
- For a plane wave travelling along unit vector n, $B = (n \times E)/v$. Both fields are perpendicular to propagation and to each other. Phase $kz - \omega t$ propagates toward +z, while $kz + \omega t$ propagates toward −z. If E is along x+y and propagation is +z, B is along −x+y, because $z \times x = y$ and $z \times y = -x$. The Poynting vector $S = E \times H$ ($E \times B/\mu$ in this medium) points along energy transport. In the radiation zone of an accelerating charge, energy flux is outward from the source, not necessarily in the instantaneous direction of charge motion.
- Maxwell's divergence equation $\nabla \cdot B = 0$ gives continuity of the normal B component across an interface: a thin pillbox has no magnetic charge inside. The tangential H jump is related to surface current; it need not vanish. Under an ideal superconducting Meissner-state model with zero interior B, the exterior normal component at the boundary must therefore be zero, leaving any nonzero exterior B tangent to the surface. This conclusion is conditional on the stated zero-interior model; it does not follow by assuming all exterior field is zero. For electrostatics, normal D can instead jump by free surface charge.

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■ Checked example

A nonmagnetic medium with εr=9 has wave speed c/3=1.0×10^8 m/s. For a +z wave with E proportional to x+y, magnetic direction is −x+y and E×B points +z. At a plane Meissner boundary with zero interior B, an exterior field may be tangential but cannot have a nonzero normal component.

■ Common error

Displacement current uses changing electric flux. Zero interior B fixes the exterior normal component, not every exterior component. Distinguish wave propagation from particle motion.

WO.1 · Acoustic Doppler echoes and standing-wave boundaries

- For sound in a stationary medium, separate source motion from observer motion. A source approaching at speed u_s compresses wavefront spacing and produces received frequency f c/(c−u_s) at a stationary observer. An observer approaching a stationary source at speed u_o meets more wavefronts per second and measures f(c+u_o)/c. Speeds are measured relative to the medium; the acoustic source and observer formulas are not symmetric under exchanging roles. Receding motion reverses the appropriate sign. These expressions assume subsonic motion along the propagation line.
- For a siren moving toward a stationary reflecting wall with speed u, the wall first receives f_wall=f c/(c−u). Reflection from a stationary wall preserves frequency in the medium frame. The moving driver then approaches the returning wavefronts and receives f_echo=f_wall(c+u)/c=f(c+u)/(c−u). At small u/c the fractional shift is approximately 2u/c, but the exact expression has different numerator and denominator. A moving reflecting surface needs its own Doppler step; do not reuse the stationary-wall result blindly.
- At an ideal open pipe end, air displacement is an antinode and pressure variation is a node. At a rigid closed end, displacement is a node and pressure is an antinode. For both ends open, length L contains n half-wavelengths, giving f_n=nc/(2L), n=1,2,… . For one end closed, it contains an odd number of quarter-wavelengths, giving f_n=(2n−1)c/(4L). Real pipes may need end corrections; the ideal GRE model uses the stated length without inventing an adjustment.
- Closing one end of a previously both-open pipe halves its fundamental and leaves only odd multiples of that new fundamental. Its old frequencies were integer multiples of c/(2L), which are even multiples of c/(4L), so none is an allowed frequency of the new ideal one-closed spectrum. This differs from simply deleting even harmonics while retaining the old fundamental. When the medium is unchanged, wave speed is unchanged; frequency and wavelength change together to satisfy the new boundary geometry.

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■ Checked example

A 500 Hz siren approaches a fixed wall at 10 m/s in air with c=340 m/s. The echo heard by the driver is 500 · 350/330=530.3 Hz. For a 0.85 m ideal pipe in the same air, the both-open fundamental is 200 Hz, while the one-closed spectrum begins 100, 300, 500 Hz. None of the original 200, 400, 600 Hz lines is retained.

■ Common error

An echo needs both Doppler factors. Closing a pipe changes the fundamental; odd harmonics are counted from the new fundamental, not the old one.

WO.2 · Optical instruments, path differences and refraction

- For an ideal astronomical telescope adjusted for relaxed viewing at infinity, the objective forms its focal-plane image and the eyepiece collimates the emerging light. Lens separation is f_objective+f_eyepiece; angular magnification magnitude is f_objective/f_eyepiece. The usual two-converging-lens telescope produces an inverted image, represented by a negative signed angular magnification under a consistent convention. A finite final-image distance changes lens separation; do not use the infinity adjustment without checking the condition.
- Resolving power R=$\lambda/\Delta\lambda$ describes the smallest distinguishable wavelength separation near λ. For an ideal diffraction grating in order m with N illuminated slits, R=mN. This is a resolution criterion, not simply the angular position formula d sinθ=mλ. Increasing illuminated slit count sharpens the principal peaks; increasing slit spacing mainly changes their angular locations. A measured λ=600 nm and $\Delta\lambda$=3 nm gives R=200, independent of converting both lengths to metres because their ratio uses matching units.
- Michelson interference depends on the optical path difference between the arms. If one arm contains a gas cell of geometric length L and index changes from n to 1 upon evacuation, a double passage changes path by 2L(n−1). If N fringes pass, the magnitude of this change is Nλ, so n−1=Nλ/(2L). For a moving mirror instead, displacement Δx gives path change 2Δx. Fringes count phase cycles; one fringe is one wavelength of optical path, not one wavelength of mirror motion.
- Snell's law n1 sinθ1=n2 sinθ2 uses angles from the local surface normal. Reflection has equal incoming and outgoing angles from that same normal. In a rectangular block, normals of neighbouring faces are perpendicular: an incidence angle measured from one normal is complementary to the angle of that same ray from the other. Draw the ray and normals before substitution. For incidence from index n into air, total internal reflection occurs only when sinθ>1/n; equality is the critical grazing case. A stated partly refracted ray must satisfy the transmission condition.

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■ Checked example

A relaxed-view telescope has lens separation 80 cm and eyepiece focal length 16 cm. Its objective focal length is 64 cm and magnification magnitude is 4. Evacuating a 5 cm gas cell causes 80 fringes at wavelength 500 nm: n−1=80 · 500×10^{-9}/(2 · 0.05)=0.0004. The initial gas index is 1.0004, not 1.0008.

■ Common error

Use optical path, not just geometric distance. Refraction angles are measured from the normal of the actual surface, and telescope focal-length addition assumes a final image at infinity.

WO.3 · Fourier symmetry and wave superposition

- For a sufficiently regular real 2π-periodic function, write f(x)=a0/2+Σ[a_n cos(nx)+b_n sin(nx)]. The coefficients are a_n=(1/π)∫from−πtoπ f(x)cos(nx)dx and b_n=(1/π)∫from−πtoπ f(x)sin(nx)dx, with a0 using n=0. Cosines and sines are orthogonal on a full period. The mean is a0/2, not a0; retain the constant term even when all nonzero-frequency coefficients of one family vanish.
- If f is even about the chosen origin, f(x)sin(nx) is odd and its symmetric integral is zero, so all b_n vanish. If f is odd, its mean and all a_n vanish. Symmetry depends on the origin: shifting the same physical signal can mix sine and cosine coefficients while leaving its harmonic frequencies unchanged. A nonnegative triangular waveform symmetric about x=0 can therefore have cosine harmonics and a nonzero mean without any sine harmonics.
- For the 2π-periodic extension of f(x)=|x| on [−π,π], the mean is π/2. Integrating x cos(nx) by parts on [0,π] gives a_n=2[(-1)^n−1]/(πn^2), so even-n cosine coefficients are zero and odd-n coefficients are −4/(πn^2). All sine coefficients are zero. The 1/n^2 decay reflects a continuous function with a slope discontinuity. A jump discontinuity often gives slower coefficient decay and partial-sum overshoot; do not infer the same convergence behaviour for every waveform.
- A Fourier sum adds amplitudes with their phases. For equal coherent monochromatic amplitudes A meeting with phase difference φ, resultant squared amplitude is 2A^2(1+cosφ), so intensity is 2I0(1+cosφ). It is 4I0 in phase and zero at $\varphi=\pi$. Incoherent averaging removes the cross term, yielding 2I0. Harmonics at different frequencies can construct a shape over time; their instantaneous sum is not the sum of their individual intensities. Identify whether the question concerns a waveform, time-average power or coherent interference.

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■ Checked example

For f(x)=|x| periodically extended, the first cosine approximation is $\pi/2-(4/\pi)\cos x$. It is even and has zero sine coefficients. At x=0 it gives $\pi/2-4/\pi\approx0.298$, which approaches the exact 0 as more odd cosine terms are added. Two equal coherent waves at phase difference $\pi/2$ have intensity 2I0; their fields do not cancel completely.

■ Common error

Even symmetry removes sine coefficients, not the constant term or every cosine harmonic. Check the symmetry origin and whether fields are coherent before adding intensities.

TS.1 · Gas processes, entropy and reversible cycles

- Use $\Delta U=Q-W$, where Q is heat entering the system and W is work done by it. Quasistatic boundary work is $\int P\,dV$, positive during expansion. For a fixed amount of ideal gas, internal energy depends only on temperature. At constant volume W=0, so $\Delta U=Q$; for an isothermal ideal-gas process $\Delta U=0$, so Q=W. Adiabatic means Q=0, not constant temperature. Over a cycle, the state returns and $\Delta U=0$, so net Q equals net W even though individual legs have different heat transfers.
- For reversible isothermal expansion of an ideal gas, PV is constant. With constant heat capacities, a reversible adiabatic path obeys $PV^{\wedge}\gamma$=constant, with $\gamma=Cp/Cv>1$; through the same initial state its pressure falls faster as volume increases. An isobaric path is horizontal on a P–V plot. A clockwise closed cycle has positive $\oint P\,dV$; reversing direction changes the sign. For a rectangular loop the magnitude is $\Delta P\Delta V$, and a triangular loop has half the corresponding bounding-rectangle area. Use Pa and m^3 for joules, not an unconverted litre value.
- For reversible heat transfer, $dS=\delta Q_rev/T$; entropy is a state function while heat is path dependent. A reversible system-plus-environment process has zero total entropy change, but the system's entropy alone can increase or decrease. Irreversible spontaneous processes produce nonnegative total entropy. A reversible isothermal expansion increases the gas entropy by $nR\ln(V2/V1)$; the reservoir loses the same amount. Reversibility does not require constant system temperature, zero work or zero internal-energy change.
- A reversible engine between TH and TC has efficiency $W/QH=1-TC/TH$. A reversible refrigerator instead has $COP=QC/W=TC/(TH-TC)$, and a heat pump has $QH/W=TH/(TH-TC)$. Use absolute kelvin temperatures. On a reversible T–S diagram, heat magnitude along an isotherm is T times the entropy change; the Carnot rectangle's area is the work magnitude. Refrigeration moves heat from cold to hot by consuming work, reversing the engine cycle. COP can exceed one without violating conservation because the moved heat is not supplied solely by the work.

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■ Checked example

A clockwise P–V rectangle spans 1×10^5 to 3×10^5 Pa and 0.001 to 0.004 m^3. Work and net entering heat are 600 J. Reversing it gives −600 J. A Carnot refrigerator operates at TC=280 K and TH=300 K with entropy transfer 2 J/K: QC=560 J, QH=600 J, work input=40 J and COP=14.

■ Common error

Zero total entropy production does not mean zero system entropy change. A refrigerator uses TC/(TH−TC), not the engine efficiency formula.

TS.2 · Speed distributions and statistical ensembles

- For a continuous speed probability density f(v), f(v)dv approximates probability in a narrow interval and ∫from0to∞ f(v)dv=1. The density has inverse-speed units and is not itself a probability. If P(v)dv counts particles in that interval, its area is the particle count N and f=P/N. A trapezoid rising over width v0, remaining flat over width 2v0, then falling over width v0 has area 3av0 for height a. Normalise by area before computing a mean or interpreting a plotted height.
- In an isotropic equilibrium gas without bulk flow, each velocity-component distribution is symmetric, so ⟨v_vector⟩=0. Speed v=|v_vector| is nonnegative and has positive mean. For a classical ideal gas, the Maxwell speed density is proportional to $v^2\exp[-mv^2/(2kBT)]$. Its mode is sqrt(2kBT/m), its mean is sqrt(8kBT/(πm)) and its rms speed is sqrt(3kBT/m). These are three different quantities. The v^2 factor makes speed density zero at v=0, even though the velocity-vector density is largest at the zero vector.
- For a continuous distribution, the probability of exactly one specified speed is zero; nonzero probabilities refer to intervals. This statement does not imply there are no particles with arbitrarily small speeds, or that a finite-resolution detector cannot record a zero bin. Temperature changes the scale of the Maxwell distribution: characteristic speeds are proportional to sqrt(T/m). Doubling temperature does not double the speed, and heavier particles are slower on average at the same temperature.
- In a canonical ensemble at temperature T, a state of energy E_i has weight exp(−E_i/(kBT)). Divide by partition function Z=Σexp(−E_i/(kBT)) to obtain probabilities. If an energy level has degeneracy g_i, its level probability is g_i exp(−E_i/(kBT))/Z. Equal energy per state does not imply equal probability per level when degeneracies differ. For a ground level of degeneracy 1 and an excited level of degeneracy 2 at Δ=kBT ln2, the excited total weight is 2e^(−ln2)=1: the two levels each have probability 1/2. Classical Maxwell–Boltzmann assumptions differ from Bose–Einstein and Fermi–Dirac quantum occupations.

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■ Checked example

A count-density trapezoid has N=90 particles, v0=2 m/s and a plateau width 2v0. Its area is 3av0, giving a=15 particles per unit speed. For the two-level ensemble with degeneracies 1 and 2 and gap kBT ln2, Z=2 and excited-level probability is 1/2. Each individual excited state has probability 1/4.

■ Common error

A density height is not a probability. Mean velocity, mean speed and mode differ; include degeneracy before normalising level probabilities.

LM.1 · Counting statistics, uncertainty and dimensional models

- For N independent incident particles each detected with probability p, count X is binomial: mean Np and variance Np(1−p). The standard deviation is sqrt[Np(1−p)], not the variance itself. With large N and small p, a Poisson approximation has mean λ=Np and variance λ, so deviation sqrt(λ). For N=200 and p=0.05, the exact mean is 10 and deviation sqrt(9.5)≈3.08, while Poisson gives sqrt(10)≈3.16. Neither model promises a fixed count. Correlated detections or dead time can invalidate independence.
- For a smooth measured function y(x1,…), linearise changes using its partial derivatives. Independent small standard uncertainties combine in quadrature: $\sigma_y^2 \approx \Sigma(\partial y/\partial xi)^2 \sigma_i^2$. Correlated inputs require covariance cross terms. For y=x^a, fractional standard uncertainty is approximately |a|σ_x/|x|. Thus kinetic energy K=½mv^2 with negligible mass uncertainty has fractional uncertainty twice that of speed. With independent mass uncertainty, combine $(\sigma_m/m)^2+(2\sigma_v/v)^2$. These approximations need small errors and a suitable local linear model.
- Random scatter measures precision; a common calibration bias affects accuracy and does not disappear by averaging repeats. For independent repeated readings, the standard uncertainty of the mean falls as 1/sqrt(n), but shared systematic error does not. The uncertainty of a physical spread and uncertainty of its estimated mean are different quantities. State whether a quoted percentage is a standard uncertainty, confidence interval or worst-case bound. Summing absolute contributions is a conservative bound, not the independent-standard-error quadrature rule.
- Dimensional analysis equates powers of mass, length and time, rather than numerical sizes. For a Planck-length form G^a $\hbar$^b c^d, dimensions are [G]=L^3/(MT^2), [$\hbar$]=ML^2/T and [c]=L/T. Requiring length gives −a+b=0, 3a+2b+d=1 and −2a−b−d=0. Hence a=b=1/2 and d=−3/2, so length is sqrt(G$\hbar$/c^3). Dimensional analysis cannot determine an arbitrary dimensionless coefficient or prove that the chosen constants are physically sufficient. Reject a

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formula with wrong units before inserting numbers.

■ Checked example

For N=200 and p=0.05, expected detector count is 10 with standard deviation sqrt(9.5). If speed has 4% small standard uncertainty and mass uncertainty is negligible, kinetic-energy uncertainty is approximately 8%. The dimensional exponents (1/2,1/2,−3/2) give zero mass power, one length power and zero time power for sqrt($G\hbar/c^3$).

■ Common error

Do not confuse mean count with guaranteed count, variance with standard deviation, or a random standard error with a calibration bias. Dimensional consistency is necessary but not sufficient.

RA.1 · Relativistic lifetime, energy and Doppler shift

- For relative speed v, define β=v/c and γ=1/sqrt(1−β^2). Proper time is measured along the particle's worldline by a clock at rest with it. If its proper mean lifetime is τ0, its laboratory mean lifetime is $\gamma\tau$0 and mean travel distance is v$\gamma\tau$0 at constant speed. The decay is statistical: mean lifetime is not a guaranteed decay time for each particle. At β=0.8, γ=5/3. Compute spacetime events consistently in one frame; proper time and coordinate time are different quantities.
- Total energy is E=γmc^2, momentum p=γmv and invariant $E^2-p^2c^2=m^2c^4$. When E and pc are expressed in the same energy units, find mc^2=sqrt($E^2-(pc)^2$), not E−pc. The nonnegative square root is required for positive rest mass. For E=13 GeV and pc=12 GeV, rest energy is 5 GeV and mass is 5 GeV/c^2. A massless particle has E=pc but need not have zero energy or momentum. These relations concern isolated-particle four-momentum, not classical mv at relativistic speed.
- Work accelerating a particle from rest equals kinetic energy K=E−mc^2=(γ−1)mc^2. At β=0.6, γ=1.25 and K=0.25mc^2, while the classical ½mv^2 gives 0.18mc^2. Classical kinetic energy is the low-speed expansion and becomes inaccurate near c. Finite acceleration work increases γ rather than allowing a massive particle to reach or exceed c. Keep total energy, rest energy and kinetic energy distinct when interpreting answer units.
- For purely longitudinal relative recession in special relativity, wavelength ratio r=λ_observed/λ_emitted=sqrt((1+β)/(1−β)). Rearranging gives β=(r^2−1)/(r^2+1). A ratio r=2 gives β=3/5, not c times r−1 from a low-speed approximation. Blueshift uses r<1 and a negative recession parameter under this convention. The formula assumes the shift is entirely kinematic; cosmological expansion or gravitational redshift requires a different model. State the question's stipulated model before interpreting a spectral ratio.

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■ Checked example

A particle has proper mean lifetime 3 μs and moves at 0.8c. Its lab lifetime is 5 μs and mean travel distance is $0.8 \cdot 3\times10^{8} \cdot 5\times10^{-6}$=1200 m. A longitudinal wavelength ratio 2 implies recession speed 0.6c. A particle with total energy 13 GeV and pc=12 GeV has rest energy 5 GeV.

■ Common error

Do not use the proper lifetime as laboratory time, subtract pc from E to find rest mass, or apply the classical Doppler approximation to a large wavelength ratio.

AT.1 · Photoelectrons, reduced mass and atomic excitation

- Photoelectric maximum kinetic energy is Kmax=h$\nu-\phi$ for photon frequency above threshold ν0=ϕ/h. A stopping-potential magnitude satisfies eVs=Kmax, so Vs is linear in ν above threshold. Increasing intensity at fixed frequency increases the available photon count and usually photocurrent, not maximum photoelectron energy. Below threshold the simple one-photon model emits no photoelectrons regardless of intensity. Work function is a property of the surface, not proportional to the illumination frequency.
- Characteristic X rays result when an electron fills an inner-shell vacancy, emitting a photon equal to the shell energy difference. Their sharp lines depend on target atoms. Bremsstrahlung arises from deceleration of energetic electrons in nuclear electric fields and produces a continuous background, with an energy endpoint set by the incident electron energy. A continuous background and discrete lines can appear together; the existence of one does not exclude the other. These mechanisms differ from visible fluorescence, phonon scattering and particle capture.
- For a hydrogen-like two-body atom, replace electron mass by reduced mass μ=m_eM/(m_e+M). In the simple Coulomb model, level energies scale as $-\mu Z^2/n^2$, so spectral frequencies scale as μZ^2 and wavelengths inversely. For positronium M=m_e, μ=m_e/2 and its Rydberg constant is half the infinite-nuclear-mass value. For a heavy nucleus μ approaches m_e. In many-electron atoms, use orbital filling and Hund's rule rather than the hydrogen model: degenerate orbitals are occupied singly with parallel spins before pairing. Carbon's $2p^2$ gives two unpaired electrons and total spin S=1; oxygen's $2p^4$ has two unpaired electrons and also S=1. Filled pairs contribute zero net spin.
- In Franck–Hertz experiments, accelerated electrons lose energy through inelastic excitation once they reach an atomic threshold. Repeated current-dip or peak spacing in accelerating voltage can therefore identify the same excitation energy lost one, two or more times. A spacing ΔV corresponds to energy eΔV; peaks at 4,8,12 V need not represent three separate excited-level energies. Contact potentials and retarding fields can shift absolute peak positions, so use the spacing and stated apparatus conditions. If the excited atom returns by one photon of that energy,

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wavelength is hc/(eΔV).

Checked example

Using h=4.136×10^−15 eV · s, light at 8×10^14 Hz has photon energy 3.3088 eV. With work function 2.0 eV, Kmax=1.3088 eV and stopping magnitude 1.3088 V. Excitation spacing 4 V gives 4 eV and, using hc≈1240 eV · nm, a 310 nm return photon. Positronium spectral wavelengths are twice the infinite-mass hydrogen values at matching transitions.

Common error

Current intensity and stopping voltage answer different questions. Include reduced mass and orbital degeneracy, and use Franck–Hertz spacing rather than treating every peak as a new energy level.

QM.1 · Scattering, degeneracy and Pauli operators

- For constant potential V and energy E>V, the spatial solutions are travelling factors exp(±ikx), with k=sqrt[2m(E−V)]/ħ. At finite steps with the same particle mass, wavefunction and its first derivative are continuous. In a left-incident scattering problem with no incoming beam from the right, the far-right solution contains only the right-travelling factor Ae^(ikx). If a finite well returns to the original external potential, the transmitted external wave number equals the incident one, even though the interior wave number differs. Exponentially decaying solutions describe E<V regions, not every potential well.
- For a step from V1 to V2 with both regions classically allowed, write incident-plus-reflected amplitude in region 1 and transmitted amplitude in region 2. Continuity gives r=(k1−k2)/(k1+k2) and t=2k1/(k1+k2). Reflection probability is R=|r|^2; transmission is T=(k2/k1)|t|^2 because probability current depends on wave number. Thus R+T=1 for a lossless step. Do not add raw squared transmitted amplitude to R without its current factor. With k2=3k1, R=1/4 and T=3/4 despite a downward step.
- A three-dimensional isotropic oscillator separates into x,y,z modes with nonnegative integers n_x,n_y,n_z. Its energy is (N+3/2)ħω, where N=n_x+n_y+n_z. For a spin-zero distinguishable single particle, the spatial degeneracy is the number of such triples: (N+1)(N+2)/2. This counts different assignments, not just different unordered partitions. For N=2, the six states are permutations of (2,0,0) and (1,1,0). For N=3, the degeneracy is 10. Additional spin or identical-particle constraints would change the counting problem.
- Pauli matrices are σ_x=[[0,1],[1,0]], σ_y=[[0,−i],[i,0]], σ_z=[[1,0],[0,−1]]. Each squares to identity. Direct multiplication gives σ_xσ_y=iσ_z and σ_yσ_x=−iσ_z, so distinct Pauli matrices anticommute. Cyclic products x→y→z have positive i; reversing order changes the sign. The general identity is

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σ_iσ_j=δ_ijI+i$\Sigma\varepsilon$_ijkσ_k. Matrix order matters: σ_xσ_z=−iσ_y, not iσ_y or simply σ_y. These dimensionless matrices become spin operators S_i=$\hbar\sigma$_i/2, adding physical units and factors.

Checked example

At a lossless step with k2=3k1, r=−1/2 and t=1/2. R=1/4, T=3 · 1/4=3/4 and their sum is one. An isotropic spin-zero oscillator at E=(9/2)$\hbar\omega$ has N=3 and degeneracy 10. Multiplying σ_x by σ_y gives diag(i,−i)=iσ_z.

Common error

Transmission probability needs the wave-number current ratio. Oscillator degeneracy counts ordered mode triples. Do not reverse Pauli matrix order without changing its sign.

SN.1 · Nuclear binding, particle families and Hall carriers

- Binding energy B is the energy needed to separate a nucleus into its constituent free nucleons, equal to the corresponding mass deficit times c^2. Binding energy per nucleon B/A is the relevant comparison across different mass numbers; a heavier nucleus can have greater total B but lower B/A. The broad B/A maximum is near the iron/nickel region. Light-nucleus fusion and very-heavy-nucleus fission can release energy when products have greater total binding. For a reaction, Q=(initial rest mass−final rest mass)c^2; positive Q means released energy. Always compare the actual specified isotopes and final products, not just their element names.
- Leptons such as electrons, muons and neutrinos are not made of constituent quarks in the Standard Model. A negative muon has charge −e and spin 1/2, like an electron, but has a different mass and belongs to a different flavour family; it is not a meson despite its historical name. Hadrons contain quarks: ordinary mesons have one quark and one antiquark, and ordinary baryons have three quarks. Corresponding antibaryons have three antiquarks. These elementary classification models do not imply every composite particle is a simple baryon or meson.
- Check charge, energy, momentum, baryon number and relevant lepton numbers in a proposed process. A proton's uud quarks sum to charge +e; neutron udd sums to zero. Beta-minus decay n→p+e−+antineutrino conserves electric charge and includes the antineutrino to balance lepton number and kinematics. A particle's mass alone does not classify it as a quark, lepton or hadron. Reaction thresholds may require kinetic energy beyond an exothermic rest-mass difference when other constraints apply.
- In a simple one-carrier conductor, transverse magnetic Lorentz force builds a Hall electric field until transverse carrier force balances. With a stated tensor/sign convention, Hall coefficient is R_H=1/(nq); negative q gives electron-like negative coefficient, positive q gives hole-like positive coefficient. Resistivity magnitude alone

does not reveal this sign. Set current, magnetic field and voltage directions consistently before reading raw polarity. Semiconductors with electrons and holes both contributing require a mobility-weighted two-carrier model; the simple coefficient need not equal the inverse total carrier density.

■ Checked example

If nucleus A has B=24 MeV and A=4, its B/A is 6 MeV; if nucleus B has B=80 MeV and A=10, its B/A is 8 MeV despite both having positive binding. Quark charges uud add to +e, udd to 0. In a one-carrier sample with density n and charge −e, R_H=−1/(ne); a raw Hall-voltage sign is meaningful only with fixed lead/field directions.

■ Common error

Compare binding per nucleon, not just total binding. A muon is a lepton, and Hall sign requires the declared measurement convention and carrier model.

CM.4 · Constraints, variational equations and cyclic coordinates

- A constraint removes an independent coordinate before you form the kinetic energy. For a fixed-length pendulum choose θ from the downward vertical: x=l sinθ and y=−l cosθ. Differentiating both coordinates gives $v^2=l^2\theta dot^2$, not $l^2\ \sin^2\theta\ \theta dot^2$. Thus $T=ml^2\theta dot^2/2$ and V=mgl(1−cosθ), with zero potential at the bottom. The fixed length is a holonomic constraint: it is a relation among coordinates and possibly time. A rolling velocity constraint needs its own analysis; do not automatically treat every constraint as a coordinate substitution.
- For ideal constraints and the usual conservative system, the stationary-action equation is d/dt(∂L/∂qdot)−∂L/∂q=0 with L=T−V. The variations vanish at the two time endpoints. A pendulum gives $\partial L/\partial\theta dot=ml^2\theta dot$ and $\partial L/\partial\theta=-mgl\ \sin\theta$, so $ml^2\theta ddot+mgl\ \sin\theta=0$. The momentum derivative acts on every time-dependent factor: for a varying radius, $d(mr^2\theta dot)/dt$ contains 2mr rdot θdot. The variational method does not mean mechanical energy is conserved in a time-dependent system.
- For planar central motion, $L=m(rdot^2+r^2\theta dot^2)/2-V(r)$. The angle θ is cyclic because L has no explicit θ dependence, even though L depends on θdot. Its canonical momentum $p\theta=mr^2\theta dot$ is constant. The radial equation is $m\ rddot=mr\theta dot^2-dV/dr$. The first term is part of the coordinate acceleration, not a new outward real force in an inertial frame. Conservation of pθ makes angular speed increase as r decreases; constant angular speed is a different, externally driven situation.
- Near a stable equilibrium q0, expand V to second order and use a constant local inertia M: $V\approx V(q0)+V''(q0)(q-q0)^2/2$. Then the small-displacement frequency is

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sqrt(V″/M). For a pendulum sinθ≈θ in radians, so ω=sqrt(g/l); the approximation requires small amplitude. A negative V″ gives instability, not an oscillation with an imaginary measurable frequency. Check the full nonlinear equation first, then state the approximation and compare units: V″/M must have units of inverse time squared.

■ Checked example

With l=2 m and g=10 m/s^2, θddot=−5 sinθ. At θ=0.10 rad the exact acceleration is −0.4992 rad/s^2, close to −0.500 from linearisation. The small-angle period is 2π/sqrt(5)=2.810 s. For a central-force orbit, shrinking r from 3 m to 1.5 m while pθ is conserved multiplies θdot by 4, not 2.

■ Common error

A cyclic coordinate is absent from L itself; its velocity need not be absent. Do not discard the time derivative of a changing r^2 factor.

CM.5 · Accelerating and rotating reference frames

- In a translating frame with origin acceleration A, Newton's equation becomes m a′=F_real−mA. An upward-accelerating elevator therefore has N−mg=ma in the ground frame, or N−mg−ma=0 for its stationary passenger in the elevator frame. These are the same prediction. The additional term is an apparent force caused by the chosen accelerating coordinates; it is not a new contact with another body. Uniform translation with A=0 changes velocity but needs no apparent force.
- For a rotating basis, differentiating a vector adds Ω×that vector. Applying this twice gives a=a_origin+a′+2Ω×v′+Ω×(Ω×r)+Ωdot×r. Here r and v′ are measured relative to the moving origin in its rotating axes; all vectors in a calculation must be expressed in the same basis at the same instant. Move the last three rotational terms to the force side to obtain Coriolis −2mΩ×v′, centrifugal −mΩ×(Ω×r), and Euler −mΩdot×r. A constant rotation removes the Euler term, not the Coriolis term.
- Take Ω along +z, an anticlockwise platform viewed from above, and a particle at r along +x. Centrifugal force is along +x with magnitude mΩ^{2}r. If the particle moves outward with v′ along +x, then Ω×v′ is +y, so Coriolis force points −y. Reversing the relative velocity reverses Coriolis; keeping v′=0 makes it vanish. Coriolis is perpendicular to v′ and does no instantaneous work on that relative motion. Centrifugal force is outward from the rotation axis, not necessarily outward from the chosen origin in an arbitrary three-dimensional position.
- A body at rest on the platform has a′=v′=0. With constant rotation and a fixed origin, its real force must be mΩ×(Ω×r), inward, cancelling the outward apparent term in the rotating equation. If rotation changes, a tangential real force must also balance the Euler term. Always check Ω→0 and A→0: ordinary inertial Newtonian

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motion must return. Do not mix the real inward centripetal requirement with an added outward real reaction on the same body; interaction partners belong in separate free-body diagrams.

■ Checked example

For m=2 kg, Ω=2 rad/s along +z, r=1 m along +x and outward v′=3 m/s, centrifugal force is +8 N in x and Coriolis force is −24 N in y. Euler force is zero at constant Ω. If the real force is zero, a′=(4,−12) m/s^2 at this instant. A separate 10 kg passenger in an elevator accelerating upward at 2 m/s^2 has N=120 N with g=10.

■ Common error

The relative velocity v′ belongs in the Coriolis term. Using the full inertial velocity counts rotation twice.

CM.6 · Fluid continuity, pressure energy and viscous flow

- For steady flow, the mass passing successive cross-sections per unit time is equal: ρAv is constant. An incompressible liquid has nearly constant ρ, so Q=Av is the volume flow rate. Halving pipe radius quarters area and multiplies mean speed by four for the same Q. Pressure does not determine Q without a model of the rest of the system. In a stationary liquid, dp/dz=$-\rho$g with z upward; therefore pressure increases by ρgh a distance h below a surface. Use absolute or gauge pressure consistently on both sides.
- For steady, incompressible, inviscid motion along a streamline with no pump or dissipative loss, Bernoulli gives p+ρv^2/2+ρgz=constant. These three terms are energy per volume and have pressure units. At equal height, larger speed requires smaller static pressure under these assumptions. A higher outlet also uses pressure/kinetic energy to gain gravitational energy. Bernoulli along one streamline need not imply the same constant on different streamlines in rotational flow. A stagnation point has v=0 and converts local speed energy to a pressure rise in the ideal model.
- Viscosity transports momentum between neighbouring fluid layers. For steady fully developed laminar flow of a Newtonian incompressible liquid in a circular tube, Q=πR$^4\Delta$p/(8ηL). Here Δp is the pressure drop along the tube, η dynamic viscosity and L tube length. Doubling radius at fixed Δp, η and L multiplies Q by sixteen. Doubling Q at fixed geometry needs twice the pressure drop. This viscous pressure loss cannot be added to a loss-free Bernoulli equation as though nothing changes; include a dissipative pressure/head loss or use the viscous model.
- The Reynolds number Re=ρvD/η compares inertial and viscous effects using characteristic speed v and length D. For a pipe use mean speed and internal diameter, not radius. Re is dimensionless: density times speed times length has the same units as dynamic viscosity. Small Re favours viscous dominance; transition to turbulence depends on geometry and disturbances, so a single threshold is not a

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universal law. Poiseuille scaling is not safe after assuming a turbulent flow. Check volume continuity, sign of pressure change and the regime before selecting a formula.

■ Checked example

Water with ρ=1000 kg/m^3 flows horizontally from area 4 cm^2 at 1 m/s into 1 cm^2. Continuity gives v_2=4 m/s and Q=0.0004 m^3/s. Ideal Bernoulli predicts p_1-p_2=1000(16−1)/2=7500 Pa. In a separate laminar tube at fixed pressure drop, increasing radius from 1 mm to 2 mm multiplies Q by 16; this is not the same fixed-Q experiment.

■ Common error

Continuity keeps Q constant across one steady pipe; Poiseuille compares Q between different systems at a specified pressure drop. State what is held fixed before comparing radius powers.

EM.4 · Polarisation, magnetisation and material fields

- Electric polarisation P is electric dipole moment per unit volume, measured in C/m^2. Define D=ε_0E+P so that $\nabla \cdot$ D=ρ_free; the total charge, including bound charge, still appears in $\nabla \cdot$ E=ρ_total/ε_0. Bound volume charge is ρ_b=−$\nabla \cdot$ P and bound surface charge is σ_b=P $\cdot$ n, where n points outward from the material. Uniform P therefore gives no bound charge in the bulk but can give surface charges of opposite sign. A dielectric is not an ideal metal: polarisation need not make its internal electric field zero.
- For a linear isotropic dielectric, P=$\varepsilon_0\chi$_e E and D=εE with ε=ε_0(1+χ_e)=$\varepsilon_0\varepsilon$_r. These simple scalar relations assume the response is linear and ignore anisotropy, strong dispersion and nonlinear effects. In a fully filled, large parallel-plate capacitor with negligible edge effects, D normal to the plates equals the free surface-charge density. If σ_free stays fixed, increasing ε reduces E=σ_free/ε. The bound charges oppose the applied field; their magnitude is not automatically equal to the free plate charge.
- For plate area A and spacing d, C=εA/d. Disconnecting the battery fixes free charge Q: V=Q/C falls and stored energy U=Q^2/(2C) falls when a dielectric increases C. Leaving an ideal voltage source connected fixes V: Q=CV and stored energy U=CV^2/2 both rise. The source supplies energy and moving the dielectric can involve mechanical work; compare the stated electrical energy quantity rather than assuming the capacitor alone is an isolated system. At a material interface n $\cdot$ (D_2-D_1)=σ_free, whereas static tangential E is continuous. Normal E generally changes when permittivity changes.
- Magnetisation M is magnetic dipole moment per unit volume, measured in A/m. In SI, B=μ_0(H+M); for a linear isotropic response M=χ_m H and B=μ_0(1+χ_m)H. B and H have different units and roles. A long uniform solenoid has H≈nI when

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end and demagnetising effects are negligible; material response then changes B. Free surface current sets n×(H$_2$−H$_1$)=K_free, and normal B remains continuous. Ferromagnetic hysteresis and saturation cannot be represented by one constant χ_m over every field.

■ Checked example

Take a fully filled capacitor with ε_r=4 and fixed free plate density σ. Gauss's law for D gives D=σ, then E=$\sigma/(4\varepsilon_0)$ and P=$\varepsilon_0(4-1)$E=$3\sigma/4$. C rises fourfold, so fixed-Q voltage and stored energy become one quarter. With the battery connected instead, V and E=V/d stay fixed while Q and U become four times larger. Separately, H=400 A/m and χ_m=0.5 give M=200 A/m and B=μ_0(600 A/m)≈0.754 mT.

■ Common error

Use free charge in the D equation and total charge in the E equation. Do not confuse fixed Q with fixed V, or identify B numerically with H.

EM.5 · RC and RL transients, impedance and resonance

- For a resistor R in series with a capacitor C and a DC source V_s, Kirchhoff's voltage law is R dq/dt+q/C=V_s. Define V_C=q/C and τ=RC. After a switch to a constant source, V_C(t)=V_s+(V_C(0)−V_s)exp(−t/τ). An initially uncharged capacitor therefore has V_C=V_s(1−exp(−t/τ)) and charging current I=(V_s/R)exp(−t/τ). With the source removed and the resistor connected across it, V_C=V_C(0)exp(−t/τ). Capacitor voltage is continuous across the switch if there is no impulsive current; the resistor current may change instantly.
- In a series RL circuit, Kirchhoff's law gives L dI/dt+RI=V_s and τ=L/R. The current after a constant drive is I(t)=V_s/R+(I(0)−V_s/R)exp(−t/τ). Inductor current is continuous for a finite voltage; its voltage may change abruptly as switching changes dI/dt. At late times an ideal inductor in a DC circuit acts as a zero-voltage connection, while an ideal capacitor has zero DC current. Stored energies are LI^2/2 and CV_C^2/2. During decay into a resistor, the initially stored energy becomes resistor heat rather than disappearing when the source is disconnected.
- Use the declared phasor convention exp(iωt). A resistor has impedance R, an inductor iωL and a capacitor 1/(iωC)=−i/(ωC). A series RLC circuit therefore has Z=R+iX with X=ωL−1/(ωC). Divide the source voltage phasor by Z to obtain current. Its magnitude is V_rms/sqrt(R^2+X^2) when rms quantities are used. The impedance phase φ satisfies tanφ=X/R; current lags voltage for X>0 and leads it for X<0. Adding the scalar magnitudes R, ωL and 1/(ωC) loses the vector phase information.
- Series resonance occurs at ω_0=1/sqrt(LC) when X=0. Current is maximal for a fixed voltage in this ideal series model, and the voltage/current phase difference is

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zero. Mean real power is V_rms I_rms cosφ=I_rms^{2}R; ideal L and C exchange energy but dissipate no average power. This differs from transient natural frequency: a damped series circuit can oscillate at sqrt(1/(LC)−(R/(2L))^2) when underdamped. Large resistance removes such free oscillation but does not change the condition X=0 of the ideal driven series impedance. State whether a question asks for a step response, free decay or sinusoidal steady state.

Checked example

For R=2000 Ω, C=0.001 F, V_s=12 V and V_C(0)=0, τ=RC=2 s. At t=τ, V_C=12(1−e^(−1))=7.585 V and I=(12/2000)e^(−1)=2.207 mA. A separate RL circuit with L=0.4 H, R=2 Ω and V_s=10 V has τ=0.2 s and I(τ)=5(1−e^(−1))=3.161 A. For series R=10 Ω, L=0.1 H, C=0.001 F and V_rms=20 V, ω_0=100 rad/s; resonance gives I_rms=2 A and mean power 40 W.

Common error

RC has τ=RC, while RL has τ=L/R. Do not confuse current amplitude with rms current, or resonance of a driven impedance with damped free-oscillation frequency.

WO.4 · Polarisation, analyser chains and phase

- For propagation along z, a transverse electric field can have x and y components. Linear polarisation means the field oscillates along one fixed line; an ideal analyser transmits the projection along its axis. Field amplitude becomes E cosθ, so intensity becomes I cos$^2\theta$. Here θ is between the incoming polarisation and that analyser, and I is the intensity immediately before it. An unpolarised beam is a statistical mixture of transverse orientations; an ideal first polariser passes half its mean intensity. The outgoing beam is then linearly polarised along the filter axis. Do not apply a new factor of one half at every later filter, or use one angle to the original source for the whole chain.
- After each analyser, update both the intensity and the polarisation direction. For an initially unpolarised beam and ideal axes α_1, α_2, α_3, the final intensity is (I_0/2)cos^2(α_2−α_1)cos^2(α_3−α_2). Two crossed filters transmit zero in this model; a middle oblique axis changes the direction before the final projection. This increase relative to the crossed pair does not create energy: every step still has transmission between zero and one. Real filters have absorption, imperfect extinction and wavelength dependence. Use the ideal law only when those losses are excluded or separately specified.
- Write E_x=A cosωt and E_y=B cos(ωt+δ) at a fixed point. Equal or opposite phases give a line, including a line at an oblique angle; equal nonzero amplitudes and a quarter-cycle phase difference give a circle. Unequal amplitudes at quarter-cycle phase give an ellipse. A quarter-wave plate adds a relative phase of π/2 between its principal axes under its design conditions. A linear input at 45° supplies equal components, so the outgoing field can be circular. A linear input along a

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principal axis has only one component and remains linear. For circular input, every ideal linear analyser passes half the intensity; handedness requires a declared viewing direction and phase convention.

- For incidence from transparent nonmagnetic medium n_1 into n_2, Brewster's angle measured from the normal satisfies $\tan\theta_B = n_2/n_1$. At that angle, the reflected p component, whose electric field lies in the plane of incidence, vanishes in the ideal dielectric model. Reflected light from unpolarised input is then s polarised, perpendicular to that plane. Snell's law gives a refracted angle complementary to θ_B. This is distinct from the critical-angle condition $\sin\theta_c = n_2/n_1$, which needs $n_1 > n_2$ and concerns total internal reflection. Brewster reflection does not imply that the entire incident intensity is reflected or that the transmitted beam is completely polarised.

Checked example

Unpolarised $I_0 = 80$ W/m^2 passes axes 0°, 45°, 90°. Intensities are 40, 20 and 10 W/m^2; without the middle filter the crossed pair would give zero. Equal $E_x = 3\cos\omega t$ and $E_y = 3\sin\omega t$ trace a circle with $E_x^2 + E_y^2 = 9$. From $n_1 = 1$ to $n_2 = 1.5$, $\theta_B = \operatorname{atan}(1.5) = 56.31°$ and refraction is 33.69°; the reflected beam is s polarised.

Common error

Malus intensity uses $\cos^2$ of the angle to the immediately preceding polarisation; a phase plate changes relative phase, while a polariser removes a field component.

WO.5 · Diffraction envelopes and missing interference orders

- For a uniformly illuminated slit of width a, far-field contributions across its opening arrive with a phase gradient $k \sin\theta$. Add their complex amplitudes before squaring: the normalised integral over x from $-a/2$ to $a/2$ is $\sin\beta/\beta$, where $\beta = \pi a \sin\theta/\lambda$. The intensity ratio is $(\sin\beta/\beta)^2$. At $\theta = 0$ take the limit $\sin\beta/\beta \to 1$, rather than calling the centre undefined or dark. Zeros occur at $a \sin\theta = m\lambda$ with nonzero integer m. The central peak lies between the first zeros and is twice as wide as one adjacent zero-to-zero interval in $\sin\theta$. Side-peak maxima are not exactly halfway between their zeros.
- At small angles on a distant screen L away, $y \approx L\theta$ and $\sin\theta \approx \theta$ give first zeros $y \approx \pm L\lambda/a$. Thus the central width is $2L\lambda/a$. Use metres consistently: a millimetre slit and a nanometre wavelength differ by six powers of ten. For a wider angle, use $\theta = \operatorname{asin}(m\lambda/a)$ and $y = L\tan\theta$; the small-angle y expression then becomes inaccurate. Far-field conditions need nearly parallel rays from different parts of the aperture, for example L much greater than a^2/λ, or the equivalent focal-plane arrangement. Near-field Fresnel patterns cannot be assigned this intensity law blindly.
- For two coherent identical uniformly illuminated slits of width a and centre sep-

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aration d, the normalised pattern is $(\sin\beta/\beta)^2\cos^2\alpha$, with $\alpha=\pi d\sin\theta/\lambda$ and central intensity as the normalisation. The $\cos^2$ factor produces interference orders $d\sin\theta=n\lambda$; the single-slit factor gives the broader envelope zeros. Small-angle neighbouring interference spacing is $\lambda L/d$. Increasing d narrows fringe spacing; increasing a narrows the envelope. These are separate changes. The equal-height narrow-slit interference formula alone does not predict the diminishing brightness or missing orders of finite apertures. Incoherent sources do not maintain the same phase-dependent cross term.

- A missing order occurs when $n\lambda/d=m\lambda/a$ simultaneously, giving $n=m\,d/a$. If d/a is an integer r, orders $\pm r$, $\pm 2r$ and so on are cancelled by aperture zeros. Do not count an order at an envelope boundary as a visible bright fringe. In the central envelope, the nominal interference-order centres satisfy $|n|<d/a$; when r is an integer there are $2r-1$ such centres. For finite slit width, exact local maxima are shifted slightly by the changing envelope, so the interference-order locations are an approximation to observed peak centres. Identify what quantity is being requested before treating every $\cos^2$ maximum as an exact maximum of the product.

■ Checked example

For λ=500 nm, a=0.10 mm, d=0.50 mm and L=2 m, first envelope zeros are approximately ±10 mm, central width 20 mm and interference spacing 2 mm. Since d/a=5, nominal orders ±5 are missing. Orders −4 through +4 give nine interference centres in the central envelope. At $\sin\theta=\lambda/(2a)$, $\beta=\pi/2$, so the single-slit intensity is $4/\pi^2=0.4053$ of its central value.

■ Common error

Add field amplitudes before squaring, distinguish a from d, and exclude dark boundary orders. An interference maximum alone does not guarantee a maximum of the full pattern.

WO.6 · Signed lens and mirror images

- Use the real-is-positive convention for this unit: a real object sends diverging incident rays into the element, so $s>0$; an already converging incident beam can represent a virtual object, $s<0$. A real image has $s'>0$, while a virtual image has $s'<0$. The paraxial equation is $1/f=1/s+1/s'$ and transverse magnification $M=-s'/s$. A converging lens has $f>0$ and a diverging lens $f<0$. For left-to-right light and a real object at $x<0$ relative to a lens at $x=0$, a real image lies at $x=s'>0$; a negative s' puts its virtual image on the input side. State the convention before substitution, as other signed-coordinate conventions give different-looking equations.
- For a converging thin lens, a ray parallel to the axis exits through the far focal point, and a ray through the ideal optical centre continues undeviated. Their intersection identifies a real image; backwards extensions can locate a virtual image. The paraxial approximation uses rays close to the axis and neglects thickness and

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aberrations. With s>f>0, s′ is positive and M is negative: the image is real and inverted. With 0<s<f, s′ is negative and M is positive: the image is virtual, upright and enlarged. At s=f, emerging rays are parallel and no finite image plane exists. A diverging lens with a real object produces an upright reduced virtual image.

- For a spherical mirror in the same real-is-positive convention, concave f>0 and convex f<0, with f=R/2 in the paraxial limit. A real reflected image is in front of the mirror on the incoming-light side; a virtual image is behind it. Thus a positive image distance has a different physical side for a mirror than for a transmitting lens. A parallel ray reflects through the concave focus; a ray aimed through the centre of curvature retraces its path. Convex reflected rays diverge as if from the focus behind the mirror. Use the same equation and M=−s′/s with signed values, rather than silently combining mirror geometry with a lens coordinate diagram.
- For a sequence of separated lenses, first calculate the actual image coordinate from the first lens. Relative to the next lens, decide whether the incoming rays diverge from a point before it or are still converging toward a point beyond it; these are real and virtual objects respectively. Only then assign its signed object distance and solve again. Total transverse magnification is the product of the individual signed magnifications for aligned paraxial elements. Element separation is not automatically the second object distance. Geometric real/virtual character depends on convergence, not on whether the final image is magnified. These ideal results do not model wave-optical resolution or spherical/chromatic aberration.

■ Checked example

A converging lens at x=0 has f=10 cm and a real object at x=−30 cm. $1/s'=1/10-1/30=1/15$, so s′=15 cm and M=−1/2. A second f=10 cm lens at x=40 cm receives rays diverging from x=15: $s_2=25$ cm, $s_2'=50/3$ cm and final x=170/3 cm≈56.67 cm. $M_2=-2/3$, so total M=+1/3. Separately a convex mirror f=−12 cm with s=24 cm gives s′=−8 cm and M=+1/3, an upright virtual image behind the mirror.

■ Common error

Keep signs until the geometry is interpreted; a positive mirror image lies on the incident side, while a positive transmitting-lens image lies on the outgoing side.

TS.3 · Thermal transport, calorimetry and expansion

- For one-dimensional steady conduction through a uniform slab, Fourier's law gives heat flux q_x=−κ dT/dx. With constant conductivity κ, area A, length L and hot-to-cold temperature difference ΔT, heat rate is Qdot=κAΔT/L. Define thermal resistance R_th=L/(κA), measured in K/W; temperature drop is Qdot R_th. Series layers without internal sources carry the same Qdot and their resistances add. Parallel paths at the same endpoint temperatures have heat rates that add, so inverse resistances add. Heat flux is rate per area, W/m^2, and need not match

between layers of different area. Interface contact resistance and heat leakage must be included if specified.

- Heat capacity C=dQ/dT depends on the thermodynamic path and is not the same as specific heat c per mass. For a material interval without a phase transition, Q=$\int$mc(T)dT, or mcΔT when c is constant. In an isolated calorimeter, sum the energy changes of all objects, including the container when relevant, and set the sum to zero. At a phase transition at its equilibrium temperature, added energy can change phase fraction rather than temperature: Q=mℓ uses latent heat ℓ in J/kg. First supply the sensible heat to reach the transition, then budget latent heat. Do not let a simple weighted-temperature average predict an impossible temperature when melting or freezing is part of the process.
- For a freely expanding rod and small temperature change, $\Delta L \approx \alpha L_0 \Delta T$ with linear expansion coefficient α. For an isotropic solid with small strain, each dimension acquires factor $1+\alpha\Delta T$, so $\Delta A/A \approx 2\alpha\Delta T$ and $\Delta V/V \approx 3\alpha\Delta T$. A hole expands with the surrounding material as if its missing region had expanded too. These relations assume a nearly constant coefficient and no mechanical constraint; anisotropic crystals need directional coefficients. If an elastic rod is prevented from changing length, its mechanical strain cancels thermal strain. With tensile stress positive, $\sigma \approx -Y\alpha\Delta T$ during heating, where Y is Young's modulus. This small-strain estimate requires elastic response without yielding or buckling.
- Conduction transfers energy through a temperature gradient, convection transports it with moving matter, and thermal radiation can cross a vacuum. For a surface at T facing large surroundings at T_env, a simple grey-body model gives net radiative rate $\varepsilon\sigma$_SB A($T^4-T_env^4$). Use absolute kelvin in these fourth powers, rather than Celsius values. A lumped body whose internal temperature remains nearly uniform can obey C dT/dt=−hA(T−T_env) under Newton cooling with constant h. Then the temperature excess decays with τ=C/(hA), rather than the entire Celsius or kelvin temperature decaying toward zero. This approximation requires sufficiently small internal gradients; a conduction-limited body needs a spatial temperature model.

■ Checked example

Two slabs of area 0.5 m^2 have L_1=0.02 m, κ_1=0.5 W/(m · K), L_2=0.03 m and κ_2=0.25 W/(m · K). R_1=0.08 and R_2=0.24 K/W; for boundaries 60°C and 20°C, Qdot=40/0.32=125 W. Drops are 10 K and 30 K, so the interface is 50°C and flux 250 W/m^2. Separately equal specific heats with masses 2 kg at 80°C and 1 kg at 20°C give isolated final 60°C without phase change. A free 2 m rod with α=10^−5 K^−1 heated 50 K lengthens 1 mm; if constrained and Y=100 GPa, σ=−50 MPa within the elastic model.

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■ **Common error**

Keep W separate from W/m^2, include the calorimeter or latent energy when needed, and do not use free-expansion length together with constrained-stress assumptions.

TS.4 · Partition derivatives, energy fluctuations and heat capacity

- For a system exchanging energy with a bath at T while N and V remain fixed, let β=1/(kBT). Sum Z=Σ_i g_i exp(−βE_i) over energy levels with their degeneracies. Level probability is g_i exp(−βE_i)/Z; mean energy U is the probability-weighted energy sum. Differentiating a temperature-independent spectrum gives U=−∂lnZ/$\partial\beta$. Keep the energy gap and degeneracies fixed during the derivative. Writing Δ=kBT ln3 to describe one evaluation temperature must not be read as making Δ change with T. For N independent distinguishable identical two-level subsystems, Z_total=z^N and U_total=N u. Indistinguishable particles and interactions need their own state counting rather than this product assumption.
- The Helmholtz free energy is F=−kBT lnZ. Canonical entropy follows from S=kB(lnZ+βU), equivalently −kBΣ p_j ln p_j over individual microstates. Constant-volume heat capacity is C_V=(∂U/∂T)_V,N. These quantities carry different units and describe different derivatives. Adding a constant energy offset ε to every state multiplies Z by exp(−$\beta\varepsilon$), increases U and F by ε, and leaves probabilities, entropy and heat capacity unchanged when ε is independent of T. A negative chosen mean energy is therefore not evidence of a negative heat capacity. Use the same energy reference in the probabilities and thermodynamic expressions.
- The second β derivative of lnZ gives variance Var(E)=⟨E^2⟩−U^2. For the same fixed-spectrum canonical model, C_V=Var(E)/(kBT^2), so C_V/kB=β^2Var(E). Nonnegative variance implies nonnegative C_V within these conditions. The denominator includes kB, not kB^2, when heat capacity retains its ordinary J/K units. This fluctuation formula concerns the canonical energy distribution; it does not say each particle has exactly the mean energy. For independent subsystems variances add, while means add too. Relative energy fluctuations typically shrink like 1/√N when the mean and per-subsystem variance remain finite and nonzero.
- For one ground state at E=0 and one excited state at fixed E=Δ>0, put x=Δ/(kBT). Then z=1+e^(−x), p_exc=1/(1+e^x), u=Δp_exc and C/kB=x^{2}e^x/(1+e^x)^2. At low T, excitation and heat capacity vanish exponentially. At high T, p_exc tends to one half and energy saturates, so heat capacity tends to zero again. Entropy rises from zero for the unique ground state toward kB ln2 as the two states become equiprobable. The resulting finite-temperature heat-capacity peak is specific to the finite-level model; it is not the constant classical oscillator value. Ground-state degeneracy or additional levels would change the limiting entropy and temperature response.

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■ Checked example

For levels 0 and fixed Δ at a temperature where x=ln3, z=1+1/3=4/3 and p_exc=1/4. Thus u=Δ/4, $\langle E^2\rangle=\Delta^2/4$ and Var(E)=$3\Delta^2/16$. C/kB=$3(\ln 3)^2/16$=0.226303. F=−kBT ln(4/3); S/kB=ln(4/3)+(ln3)/4=0.562335. Raising the reference by ε changes u to $\varepsilon+\Delta/4$ but preserves the variance and heat capacity.

■ Common error

Hold Δ fixed when differentiating, include degeneracies in Z, and use energy variance rather than the square of mean energy in the heat-capacity formula.

TS.5 · Quantum occupations and limits of equipartition

- For noninteracting particles in thermal and particle exchange equilibrium, use chemical potential μ and x=($\varepsilon-\mu$)/(kBT). Mean occupation of one complete state is n_F=1/(e^x+1) for fermions and n_B=1/(e^x−1) for bosons. Fermionic occupation of that state is zero or one; its mean lies between them. Bosonic occupation can exceed one. A level of degeneracy g has total mean g times the single-state occupation when its states share the same energy. Count spin as part of a complete state. The Bose denominator requires $\varepsilon>\mu$ for the ordinary finite expression, with the ground-state limit treated separately. For equilibrium photons μ=0 because photon number is not conserved; do not set μ=0 for every material particle gas.
- Count occupation patterns rather than labelling identical particles. Two identical fermions distributed among four distinct complete states have choose(4,2)=6 allowed patterns. Two identical bosons among those states have choose(4+2−1,2)=10 patterns because both may share one state. Two labelled distinguishable particles would have 4^2=16 assignments. These are different counting models, not three interchangeable answers to the same specification. If a question supplies spin degeneracy, first decide whether its stated number counts complete states or just orbital levels. A Pauli prohibition on two identical complete states does not prohibit opposite-spin fermions in one spatial orbital.
- When x is large and positive, occupation is small and both denominators are dominated by e^x: n_F≈n_B≈e^(−x), the Maxwell–Boltzmann dilute limit. At x=ln4 the means are 1/5 for fermions, 1/3 for bosons and 1/4 in the classical approximation; the difference is still significant. A large total particle number alone does not justify classical statistics: density, temperature and accessible states control occupation. For fermions at low T, states below μ become nearly occupied and those above nearly empty. At $\varepsilon=\mu$ a fermionic state has mean one half; inserting that value into the ordinary Bose formula would instead produce a divergent denominator and requires different limiting treatment.
- Classical equipartition assigns kBT/2 of mean energy to each independent quadratic term in an equilibrated Hamiltonian. A monatomic ideal gas has three

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translational terms, giving U=3NkBT/2 and C_V=3NkB/2. One classical one-dimensional harmonic oscillator has kinetic and potential terms, giving mean kBT and C=kB. For a quantum oscillator with fixed spacing $\varepsilon=\hbar\omega$, the thermal energy above its temperature-independent zero point is $\varepsilon/(e\hat{}x-1)$, now $x=\varepsilon/(kBT)$, and $C/kB=x^2e\hat{}x/(e\hat{}x-1)^2$. At high T it approaches the classical value; at low T excitation freezes out and $C\to0$. A zero-point energy $\varepsilon/2$ shifts U but not C. Molecular rotational/vibrational contributions likewise need their energy scales checked before assigning classical quadratic terms.

■ Checked example

At $x=(\varepsilon-\mu)/(kBT)=\ln4$, a single state has n_F=0.2 or n_B=1/3. A five-state degenerate level therefore has mean 1 fermion or 5/3 bosons; total fermionic level occupation can exceed one at other x because there are five distinct states. A quantum oscillator at $\varepsilon/(kBT)=\ln2$ has mean excitation energy ε and $C/kB=2(\ln2)^2=0.960906$, close to but different from the classical value one.

■ Common error

Exclude two fermions from one complete state, not from an entire degenerate energy level. State which x is used; chemical-potential occupations and fixed oscillator excitation formulas are different models.

QM.2 · Weak perturbations and degenerate subspaces

- Let $H=H_0+\lambda W$, where λ is a small dimensionless parameter and the eigenstates of H_0 are known and normalised. For a nondegenerate level n, the first-order energy correction is $\lambda\langle n|W|n\rangle$. In position space this is $\lambda\int\psi_n^*(x)W(x)\psi_n(x)dx$ for a multiplicative potential, with integration over the allowed domain. A potential value at one point is not an expectation value; the state's probability density weights the whole domain. For an infinite well $0<x<L$, $\psi_n=\sqrt{(2/L)}\sin(n\pi x/L)$. Reflection about its centre makes $\langle x\rangle=L/2$, so a weak added potential γx has shift $\gamma L/2$ for every nondegenerate well level at first order. γ has units of energy per length.
- For a perturbation odd about the centre, such as $\gamma(x-L/2)$, the probability density of an unperturbed well eigenstate is even, so its diagonal expectation vanishes. A centred quadratic perturbation $\eta(x-L/2)^2$ instead gives shift $\eta L^2[1/12-1/(2\pi^2n^2)]$, which is positive for $\eta>0$ and depends on n. Parity makes the first-order integral zero only for the specified state and operator symmetry. Off-diagonal matrix elements can still change the wavefunction. Do not turn a symmetry cancellation into a claim that every energy correction vanishes or that the original state remains exact.
- For a nondegenerate state, the leading admixture of another unperturbed state m is proportional to $\lambda W_mn/(E_n^0-E_m^0)$. The useful smallness condition therefore compares coupling matrix elements with the relevant level separations, not just

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with an arbitrary absolute energy zero. To second order, the energy correction contains $\Sigma_(m\neq n)|\lambda W_mn|^2/(E_n^0-E_m^0)$. For the lowest nondegenerate state, all these denominators are negative, so the second-order correction is nonpositive in this model. Near a degeneracy, a small denominator defeats the nondegenerate expansion; use a coupled subspace instead of dividing by zero.

- If a level of H_0 is exactly degenerate, choose an orthonormal basis within that subspace and form the Hermitian matrix of the perturbation there. Its eigenvalues are the first-order energy shifts, and its eigenvectors specify the combinations that diagonalise the leading splitting. For a two-state subspace with perturbation ε[[2,1],[1,2]], the normalised symmetric and antisymmetric combinations have shifts 3ε and ε. Reading only the two diagonal entries would incorrectly predict two shifts of 2ε. A common scalar multiple of the identity shifts both states equally and does not split their degeneracy; off-subspace couplings can matter at higher order.

Checked example

In a 2 nm infinite well, a weak γx perturbation with γ=0.05 eV/nm gives $\Delta E_n\widehat{\ }(1)=\gamma L/2=0.05$ eV. Separately, η=0.03 eV/nm^2 multiplying $(x-L/2)^2$ gives the ground shift $0.03\cdot 4[1/12-1/(2\pi^2)]=0.00392073$ eV. These are first-order results conditional on weak coupling relative to gaps. A degenerate pair with ε[[2,1],[1,2]] and ε=0.004 eV splits into shifts 0.004 and 0.012 eV; the symmetric state gets the larger shift.

Common error

Use normalised state weights, keep perturbation units, and diagonalise a degenerate block. First-order cancellation and exact invariance are different claims.

QM.3 · Exchange symmetry, spin pairs and Pauli exclusion

- For identical particles, exchanging every coordinate and spin label changes a fermionic total wavefunction by a minus sign and leaves a bosonic one unchanged. Mathematical slots 1 and 2 label arguments, not permanently distinguishable particles. For two distinct orthonormal orbitals a and b, spatial combinations are $\Psi_\pm=[a(1)b(2)\pm b(1)a(2)]/\sqrt{2}$. Orthogonality makes these combinations normalised and gives exchange eigenvalues ±1. If the two orbitals are identical, the antisymmetric combination is identically zero and the displayed symmetric formula is not correctly normalised; the double-occupation spatial state is simply a(1)a(2). Recheck normalization whenever orbitals or their overlaps change.
- Two spin-1/2 particles have one spin singlet $(\uparrow\downarrow-\downarrow\uparrow)/\sqrt{2}$ with total spin S=0; it is antisymmetric under exchange. The three triplets $\uparrow\uparrow$, $(\uparrow\downarrow+\downarrow\uparrow)/\sqrt{2}$ and $\downarrow\downarrow$ have S=1 and are symmetric. Electrons require an antisymmetric total state, so a symmetric spatial state pairs with the singlet, while an antisymmetric spatial state pairs with a triplet. Two electrons in the same spatial orbital can form the singlet, but cannot form a triplet there in this simple two-electron state. Pauli exclusion prevents

occupation of the same complete single-particle state, including spin. Opposite-spin electrons in one orbital are two different complete states; describing exclusion as one electron per orbital discards this distinction.

- For two spatial orbitals a,b and two spin states each, there are four complete single-particle states. Two identical electrons have choose(4,2)=6 occupation patterns: two double-occupation singlets, one different-orbital singlet, and three different-orbital triplets. For three spatial orbitals, there are six complete states and choose(6,2)=15 patterns. Counting ordered assignments overcounts identical particles. Spinless bosons in two orbitals instead have three occupations: both in a, one in each, both in b. These counts assume no additional energy restriction and the stated accessible single-particle states; restricting total energy or spin projection changes the allowed subset.
- An antisymmetric spatial state satisfies $\Psi_-(x,x)=0$, so its joint position density vanishes on the coincidence line. A symmetric spatial state need not vanish there. The difference arises from interference between exchanged amplitudes, even for noninteracting particles; it is not a separately imposed classical repulsive force. For a well of length L, choose $a(x)=\sqrt{(2/L)}\sin(\pi x/L)$, $b(x)=\sqrt{(2/L)}\sin(2\pi x/L)$. At $x_1=L/4$ and $x_2=3L/4$, the antisymmetric spatial amplitude is $-2/L$ and its density is $4/L^2$; the symmetric amplitude is zero. These are joint probability densities per two lengths, not dimensionless probabilities at exact points. Physical spin compatibility still decides which total electronic state uses each spatial combination.

■ Checked example

With two orthonormal orbitals a,b, the two-electron state $\Psi_+\chi_-$singlet is antisymmetric overall because $(+1)(-1)=-1$. $\Psi_-\chi_-$triplet is also allowed. If both electrons use a, the spatial product a(1)a(2) is symmetric, so only the singlet spin factor is allowed. In a well at positions L/4 and 3L/4, $\Psi_-=-2/L$ gives density $4/L^2$ while $\Psi_+=0$; each is normalised over the complete two-coordinate domain, rather than interpreted as a probability at one exact pair.

■ Common error

Exchange all spatial and spin arguments. Do not confuse antisymmetric space with antisymmetric total state, or use the distinct-orbital $\sqrt{2}$ factor for two identical orbitals.

QM.4 · Finite-well bound states and hydrogenic quantum numbers

- Declare a symmetric well $V=-V_0$ for $|x|<a$ and $V=0$ outside, with $V_0>0$ and the same mass throughout. A bound energy lies between $-V_0$ and zero. Inside, $k=\sqrt{[2m(E+V_0)]}/\hbar$ gives oscillatory solutions; outside, $\kappa=\sqrt{(-2mE)}/\hbar$ gives decaying tails. Reject growing exponentials to obtain a normalisable state. Reflection

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symmetry permits even interior cos(kx) or odd sin(kx) states. At finite boundaries without delta interactions, both ψ and ψ' are continuous. Even matching gives k tan(ka)=κ; odd matching gives −k cot(ka)=κ. A finite well does not require ψ to vanish at its edges as an infinite wall does.

- Introduce z=ka and ρ=a$\sqrt{(2mV_0)/\hbar}$. Then κa=$\sqrt{(\rho^2-z^2)}$ and 0<z<ρ. Solve z tan z=$\sqrt{(\rho^2-z^2)}$ for an even state, or −z cot z=$\sqrt{(\rho^2-z^2)}$ for an odd one, on intervals with the appropriate sign and away from tangent poles. Energy follows as $E/V_0=z^2/\rho^2-1$. For ρ=1, the ground root lies in 0<z<1 and the even equation is equivalent there to z=cos z. Bisection or a converged root finder gives z≈0.739085 and $E/V_0\approx-0.453753$. Since $\rho<\pi/2$, no odd bound-state branch fits. In one dimension an attractive square well has an even bound ground state however shallow it is; an E=0 threshold tail is not square integrable.
- For an even bound state write ψ=A cos(kx) inside and ψ=A cos(ka)exp[−κ(|x|−a)] outside. Continuity sets the relative tail amplitude; integrate $|\psi|^2$ over both the interior and tails to determine A. Probability outside the well is nonzero and depends on its depth, width and the selected state. Bound states have discrete energies and normalisable decaying asymptotes. A scattering state with E>0 has propagating external waves and is normalised or interpreted using a continuum/flux convention. A decaying tail is not evidence that the state violates energy conservation, and raw tail amplitude is not the outside probability until it has been integrated and normalised.
- For a one-electron Coulomb ion with nuclear charge Ze, use the nonrelativistic central-potential model and a heavy nucleus approximation unless reduced mass is specified. $E_n\approx-13.6\ Z^2/n^2$ eV; allowed orbital numbers are n≥1, l=0,…,n−1 and m_l=−l,…,l. L^2 has eigenvalue $l(l+1)\hbar^2$ and L_z=m_l$\hbar$; the ground 1s orbital has l=0, despite the historical Bohr circular-orbit rule. Spatial degeneracy at fixed n is Σ_l(2l+1)=n^2 before spin and fine-structure corrections. If ψ=R_nl(r)Y_lm with angular part normalised, radial probability in dr is $r^2|R|^2$dr, not just $|R|^2$dr. For hydrogen 1s, the radial density is $4r^2e^{(-2r/a_0)}/a_0^3$: it peaks at r=a_0 and has mean $3a_0/2$, while the three-dimensional point density is largest at the origin. Distinguish these measures before locating a most probable radius.

■ Checked example

For ρ=1, z≈0.739085 gives κa≈0.673612 and $E\approx-0.453753V_0$; the even ground state leaks beyond ±a but decays at infinity. For a one-electron Z=2 ion in n=2, $E\approx-13.6\cdot4/4=-13.6$ eV. Its allowed l values are 0 and 1, giving four spatial states before spin. In a hydrogen 1s orbital, the radial density peaks at a_0 because the r^2 shell factor offsets the decreasing point density; integrating the radial density gives $\langle r\rangle=3a_0/2$.

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■ **Common error**

Use the declared potential zero, match ψ' as well as ψ, and include the radial shell measure. A bound tail and a continuum travelling wave have different energy regimes.

AT.2 · Atomic field shifts and spectral differences

- An electron has a negative magnetic moment: μ_L=$-\mu$_B L/$\hbar$ and μ_S≈-2μ_B S/$\hbar$. With B along positive z, the interaction $-\mu \cdot$ B therefore gives H_Z=μ_B B(L_z+2S_z)/$\hbar$. μ_B≈5.788×10^{-5} eV/T. In an uncoupled basis with orbital projection m_l and spin projection m_s=±1/2, the shift is μ_B B(m_l+2m_s). Use this uncoupled rule when the magnetic interaction is large relative to the relevant spin-orbit splitting but still small relative to orbital energy gaps, so quadratic diamagnetic and orbital restructuring effects can be neglected. Large and small are comparisons of energies, not universal Tesla thresholds. In the opposite weak-field regime, m_l and m_s are not independently conserved labels for an LS-coupled eigenstate.
- When LS coupling dominates a weak magnetic perturbation, J=L+S and m_J label the multiplet. The first-order shift is μ_B B g_J m_J, with g_J=1+[J(J+1)+S(S+1)−L(L+1)]/[2J(J+1)] in the approximation g_s=2. L,S,J here are dimensionless quantum numbers. For L=1,S=1/2, J=3/2 gives g_J=4/3 and shifts −2,−2/3,+2/3,+2 times μ_B B. J=1/2 gives g_J=2/3 and shifts ±μ_B B/3. Do not substitute J=0 into the singular formula; a J=0 state has no first-order vector projection shift. Intermediate fields require a coupled Hamiltonian rather than mixing weak-field and uncoupled formulas.
- A photon's energy is the upper atomic energy minus the lower energy. Its field-induced change is therefore ΔE_γ=ΔE_upper−ΔE_lower. Count allowed transitions using the stated selection rules as well as the two level patterns. Several transitions can share one frequency; the number of split states is not automatically the number of spectral lines. In a spin-neglected orbital model, an upper L=1 triplet has shifts m_l μ_B B with m_l=−1,0,+1, and a lower L=0 state has zero shift. Electric-dipole Δm_l=0,±1 permits three photon shifts $-\mu$_B B,0,+μ_B B. This normal-triplet model does not describe every real atom or fine-structure multiplet. Polarisation and observation direction can change which components are detected.
- In a uniform electric field F along z, an electron's perturbation is +eFz for the stated electrostatic potential convention. A nondegenerate parity eigenstate has $\langle z \rangle$=0, so its first-order diagonal Stark shift vanishes, although a quadratic shift can remain. Exactly degenerate opposite-parity states instead allow an off-diagonal dipole matrix element. In an original two-state model, suppose the dipole matrix is [[0,d],[d,0]], with real d in charge×length units. The perturbation F times this matrix has eigenvalues ±dF and opposite superpositions of the two states. Changing

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a basis phase changes the off-diagonal sign but not the splitting 2|dF|. Actual near-degenerate atoms require comparison with fine structure and other small splittings before an exactly degenerate approximation is adopted; this supplied matrix is not a universal hydrogen coefficient.

■ Checked example

With μ_B=5.788×10^{-5} eV/T, B=2 T and weak-field LS-coupled labels L=1,S=1/2,J=3/2,m_J=1/2, the shift is (4/3)(1/2)μ_B B=7.7173×10^{-5} eV. In a different, uncoupled regime, m_l=1,m_s=−1/2 instead gives zero linear shift. A normal orbital L=1→0 triplet at 2 T has photon shifts −1.1576×10^{-4},0,+1.1576×10^{-4} eV. Separately, a degenerate dipole pair with d=2 e · nm and F=10^5 V/m has shifts ±0.0002 eV, a 0.0004 eV splitting.

■ Common error

Declare the coupling regime, keep the negative electron moment sign, and subtract lower-level shifts. Zero diagonal dipole expectation does not exclude degenerate mixing or quadratic shifts.

AT.3 · Thermal spectra and one-electron scaling

- For ideal thermal equilibrium radiation, Planck's wavelength spectral radiance is B_λ=2hc^2/{λ^5[exp(hc/(λk_B T))−1]}. B_λ is per wavelength interval and per solid angle, not a total power. A blackbody's hemispheric surface flux spectrum is M_λ=πB_λ; integrating over all wavelengths gives σ_SB T^4. All temperatures are absolute Kelvin. The wavelength peak obeys λ_max T≈2.898×10^{-3} m · K. A spectrum per frequency has a different peak because B_ν dν and B_λ dλ include a Jacobian; c/λ_max is not the peak frequency of B_ν. Increasing T moves the wavelength peak shorter and increases the integrated flux by T^4, not by the peak-position ratio alone.
- For an original uniform grey surface of area A and wavelength-independent emissivity ε, net radiative power to a large uniform environment is $\varepsilon\sigma$_SB A(T^4−T_env^4) under the stated view-factor assumptions. Use σ_SB≈5.670×10^{-8} W · m^{-2} · K^{-4}. Spectrally varying emissivity or incomplete surroundings requires a more detailed model. A photon at a specified wavelength has E=hc/λ, conveniently about 1240 eV · nm/λ_nm. A thermal spectrum contains many photon energies; a photon at the wavelength peak is not the mean energy of every photon. For T=3000 K, λ_max≈966 nm; its photon energy is about 1.284 eV. A surface with A=2×10^{-4} m^2 and ε=0.5 emits 459.27 W to a negligibly cold environment.
- In the historical Bohr one-electron model with a heavy nucleus of charge Ze, Coulomb force m_e v^2/r=Ze^2/(4$\pi\varepsilon_0$r^2) and angular momentum m_e vr=n$\hbar$ lead to r_n=a$_0$n^2/Z, v_n=Zαc/n and E_n≈−13.6Z^2/n^2 eV. The radius scales with n^2/Z while binding energy scales with Z^2/n^2; do not use the same charge power in both. These circular-orbit assumptions are a historical scaling model. Wave-mechanical

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angular momentum is $\sqrt{[l(l+1)]}\hbar$ with l=0,…,n−1, so a 1s orbital has zero orbital angular momentum rather than the Bohr $n\hbar$ value. Reduced-mass corrections replace m_e with μ: Coulomb radius scales as $1/\mu$ and energy as μ. Multielectron screening, large-Z relativistic corrections and fine structure are outside the simple model.

- For an emission ni→nf with ni>nf, $E_\gamma \approx 13.6Z^2(1/nf^2 - 1/ni^2)$ eV. Convert this positive difference to $\lambda \approx 1240/E_\gamma$ nm. Ionisation from n requires energy $13.6Z^2/n^2$ eV to reach the continuum zero. Absorption reverses the level ordering and needs the corresponding positive incoming photon energy. For a series ending at fixed nf, the largest bound-bound photon energy occurs as ni→∞, giving $E_{limit} = 13.6Z^2/nf^2$ and the shortest series wavelength. For one-electron helium Z=2, n=3→2 gives 7.5556 eV and $\lambda \approx 164.12$ nm. Its Bohr n=3 radius is $4.5a_0$; this radius describes the historical orbit, not a universal radial mode for every l at n=3.

Checked example

A 3000 K blackbody has wavelength peak 966 nm and peak-wavelength photon energy 1240/966≈1.284 eV. For ε=0.5,A=2×10^{-4} m^2,T_env=300 K, net grey-surface power is 459.224073 W; the cold-environment approximation gives 459.27 W. Separately, a Z=2 one-electron ion emits n=3→2: $13.6 \cdot 4(1/4-1/9) = 7.5556$ eV, $\lambda \approx 164.12$ nm. Its n=3 binding threshold is 54.4/9≈6.0444 eV, smaller than this photon energy because that photon ends at a lower n=2 level.

Common error

Declare a wavelength or frequency density, use Kelvin for fourth powers, and distinguish transitions from ionisation. A Bohr circular radius and angular momentum are not every wave orbital's radius and L.

RA.2 · Simultaneity, length measurement and velocity transformation

- Use the Lorentz transformation with declared frame motion: if frame S′ moves at +v along x relative to S, then $x' = \gamma(x - vt)$ and $t' = \gamma(t - vx/c^2)$ with $\gamma = 1/\mathrm{sqrt}(1 - v^2/c^2)$. Apply it to event separations Δx and Δt, not only to single coordinates. Two events simultaneous in S ($\Delta t = 0$) at different places have $\Delta t' = -\gamma v \Delta x/c^2$: the event further in the +x direction happens earlier in S′. Simultaneity is frame-dependent, not a bookkeeping error.
- A moving rod length must be read from endpoint positions at the same lab time ($\Delta t = 0$). Then $\Delta x' = \gamma \Delta x$ gives $L = L0/\gamma$ with L0 the rest-frame (proper) length. Reading the endpoints simultaneously in the rod frame instead does not produce the lab length; the two reading conditions answer different questions. State the frame and the reading condition before quoting any length.
- The interval $s^2 = c^2\Delta t^2 - \Delta x^2$ (mostly-minus convention, declared once) is invariant.

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$s^2>0$ is timelike: a frame exists with the events co-located, and $\Delta\tau=s/c$ is the proper time between them. $s^2<0$ is spacelike: a frame exists with the events simultaneous, and $\mathrm{sqrt}(-s^2)$ is the proper distance. $s^2=0$ is lightlike. Timelike order is the same in every frame; spacelike order can swap, and no signal can join the events because $|\Delta x/\Delta t|>c$.

- Collinear velocities add as $u=(u'+v)/(1+u'v/c^2)$; the denominator keeps $u<c$ for any $u',v<c$, and $u'=c$ gives $u=c$ exactly. Energy and momentum form a four-vector with invariant $E^2-p^2c^2=m^2c^4$: compute it in any convenient frame. Check each result in the nonrelativistic limit, where the addition rule reduces to $u=u'+v$ and the invariant reduces to the rest energy.

■ Checked example

Events A and B are 900 m apart in the lab and 2 μs apart in time ($c\Delta t=600$ m using $c=3\times10^8$ m/s). Since $600<900$ the interval is spacelike; the simultaneity frame moves at $v=c^2\Delta t/\Delta x=(2/3)c$ and the proper distance is $\mathrm{sqrt}(900^2-600^2)=670.82$ m. A 0.5c probe from a 0.5c ship has lab speed $(0.5+0.5)c/(1+0.25)=0.8c$, where $\gamma=5/3$. $E=5$ GeV with $p=3$ GeV/c gives rest energy $\mathrm{sqrt}(25-9)=4$ GeV.

■ Common error

Quoting $L=L0/\gamma$ without the simultaneous-endpoint condition, or adding velocities as $u=u'+v$ at relativistic speeds. A simultaneity difference is physical, not a mistake.

LM.2 · Measurement loading, amplifier gain and detector response

- Model the measured circuit as a Thevenin source V_th with resistance R_th and the instrument as input resistance R_m. The reading is $V_th \cdot R_m/(R_m+R_th)$, with the same sign as V_th and magnitude no greater than $|V_th|$ for positive resistances. A 10 V source with R_th=100 kΩ read by a 1 MΩ meter gives $10\times(1/1.1)=9.09$ V, a 9% low bias. A 10× oscilloscope probe raises the effective input resistance and divides the signal by ten; quote both effects.
- An ideal non-inverting op-amp stage has gain 1+R2/R1; the inverting stage has −R2/R1, with the sign carried explicitly. A first-order RC low-pass has cutoff $f_c=1/(2\pi RC)$; its amplitude ratio is $1/\mathrm{sqrt}(1+(f/f_c)^2)$ and its phase is $-\arctan(f/f_c)$. At f=f_c the amplitude is $1/\sqrt{2}$ (the −3 dB point), not one half. For R=1 kΩ and C=100 nF, $f_c\approx1.59$ kHz.
- Detector efficiency ε is detected events divided by incident events; energy resolution is quoted as FWHM/E, such as 13.2 keV on a 662 keV line, about 2%. A non-paralyzable detector with dead time τ records $m=n/(1+n\tau)$ from true rate n; inverting gives $n=m/(1-m\tau)$, which fails as $m\tau\to1$. State which dead-time model you assume; paralyzable behaviour differs.
- A linear calibration against known standards can correct offset and scale error;

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it does not remove noise or guarantee correction of nonlinearity. A calibration curve maps indicated to true values; interpolation between points assumes local smoothness. Report resolution and efficiency separately: sharp peaks with poor efficiency still miss events, and high efficiency with broad resolution still mixes nearby lines.

■ Checked example

A 12 V source with 200 kΩ internal resistance read by a 1 MΩ meter shows $12\times(1/1.2)=10.0$ V. An RC filter with f_c=1 kHz driven at 2 kHz passes amplitude $1/\sqrt{5}\approx0.447$. A counter with $\tau=10$ μs at true rate 10^4 /s records 9091 /s; a recorded 8000 /s with $\tau=20$ μs implies $n=8000/(1-0.16)\approx9524$ /s.

■ Common error

Treating the $1/\sqrt{2}$ cutoff as one half, ignoring that a finite meter reads low through the divider, or applying the non-paralyzable correction beyond its range.

LM.3 · Radiation attenuation, laser gain and cavity modes

- Heavy charged particles lose energy continuously through many small collisions, so they have an approximate range: under a constant-loss model a particle with initial energy E0 and loss rate dE/dx stops after E0/(dE/dx). Electrons straggle more and radiate. Photons instead interact in single events, so a narrow beam attenuates exponentially as $I=I0\ e^{(-\mu x)}$ with no definite maximum depth; the half-value layer is $\ln2/\mu$.
- Apply the exponential only to the stated narrow-beam geometry: $\mu=0.2$ /cm gives a half-value layer of 3.47 cm and $I/I0=e^{(-2)}\approx0.135$ after 10 cm. Multiply by detector efficiency for recorded counts: with $\mu x=\ln4$ and $\varepsilon=0.25$ the recorded fraction is $0.25\times1/4=1/16$. Build-up from scattered photons makes broad-beam shielding transmit more than the narrow-beam exponential; state the geometry.
- Stimulated emission produces a photon matching the stimulating photon in frequency, direction and phase, giving coherent amplification. It competes with absorption; net gain needs population inversion $N2/g2>N1/g1$ ($N2>N1$ for equal degeneracies). Positive-temperature equilibrium has $N2/g2<N1/g1$ by the Boltzmann factor, so a two-level system in equilibrium cannot lase continuously; practical lasers pump a third level or a metastable state.
- A linear cavity of length L supports standing modes with $L=m\lambda/2$, i.e. frequencies $\nu_m=mc/(2L)$ with spacing $c/(2L)$: for L=30 cm the spacing is 500 MHz. Gain must exceed the round-trip losses for oscillation. Interferometers such as Michelson use the same coherence: fringe counts track optical path changes, as in the gas-cell measurement.

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■ Checked example

For μ=0.2 /cm the half-value layer is ln2/0.2=3.47 cm, and three such layers pass 1/8. Under a constant 2 MeV/cm loss model a 5 MeV particle stops after 2.5 cm; a photon beam instead keeps an exponential tail. A 25 cm cavity has mode spacing c/(2L)=600 MHz.

■ Common error

Assigning photons a definite range or charged particles a single exponential law, and forgetting that equilibrium two-level populations cannot invert. Check which interaction model the beam species requires.

SN.2 · Bragg diffraction and electron-gas heat capacity

- Bragg reflection from parallel planes spaced d interferes constructively when 2d sinθ=nλ, with θ measured from the plane surface (the glancing angle), not from the normal. First order n=1 gives the smallest angle. For λ=0.154 nm and a peak at θ=20°, d=0.154/(2 sin20°)≈0.225 nm. If 2d<λ no order exists; the sine must stay at or below 1.
- In a cubic crystal the (hkl) plane spacing is d=a/sqrt(h^2+k^2+l^2) with lattice constant a; the (110) planes give a/√2. A measured Bragg angle therefore measures the lattice constant. Systematic absences from the basis (for example body-centred lattices missing h+k+l odd) carry structure information beyond the spacing formula.
- The free-electron model packs N conduction electrons into states up to the Fermi wavevector k_F=(3π^2n)^(1/3) for number density n, giving E_F=$\hbar^2$k_F^2/(2m). For n=8.5×10^{28} /m^3, k_F≈1.36×10^{10} /m, E_F≈7.05 eV and T_F=E_F/k_B≈8.2×10^4 K. Since T_F far exceeds room temperature, only a fraction of order T/T_F of the electrons are thermally excited.
- Metals therefore have two low-temperature heat-capacity terms: an electronic term γT linear in T and a lattice (Debye) term βT^3. Plotting C/T against T^2 gives intercept γ and slope β. At room temperature the lattice part approaches the classical Dulong–Petit value and dominates; the electronic term matters at low T, where the T^3 lattice term collapses faster.

■ Checked example

Cu Kα λ=0.154 nm with a first-order peak at θ=20° gives d≈0.225 nm; for cubic a=0.4 nm the (110) spacing is 0.283 nm. The free-electron count for n=8.5×10^{28} /m^3 gives E_F≈7.05 eV and T_F≈8.2×10^4 K, so at 300 K only about 0.4% of electrons are thermally active. In C=γT+βT^3, γT is electronic and βT^3 is lattice.

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■ **Common error**

Measuring θ from the plane normal instead of the plane, or assigning the whole low-T heat capacity to electrons. The sine condition and the two-term decomposition are where these calculations fail.

SN.3 · Reaction energies, activity and decay balances

- The reaction energy is Q=(m_initial−m_final)c^2; Q>0 releases kinetic energy. Track every species, and keep the mass convention consistent: tabulated atomic masses include electrons. For $\beta-$ decay and for α decay with a neutral helium product, atomic electron counts balance directly; $\beta+$ or electron capture needs explicit electron-mass terms. Use 1 u=931.5 MeV/c^2.
- Alpha decay of ^{238}U with atomic masses 238.0508 u → 234.0436 u + 4.0026 u gives Δm=0.0046 u and Q≈4.28 MeV. Fusion D+T with 2.0141+3.0161−4.0026−1.0087=0.0189 u gives Q≈17.6 MeV. A reaction also requires charge and nucleon-number balance: ^{14}N+$\alpha\rightarrow^{17}$O+p balances 7+2=9 and 14+4=18.
- Activity is A=λN with decay constant λ=ln2/T½; one becquerel is one decay per second. For T½=2 h and N=6×10^{12}, A=6×10^{12}×ln2/7200≈5.8×10^8 Bq. After three half-lives both N and A fall by 1/8. Equal activities do not imply equal atom counts: shorter half-life at fixed N gives larger A.
- In a chain A→B→C with a long-lived parent (λ_A≪λ_B), B accumulates until its activity approaches the parent activity (secular equilibrium); write the balance dN_B/dt=λ_A N_A−λ_B N_B. The binding-energy-per-nucleon curve peaks near iron, so heavy fission (about 7.6 to 8.5 MeV per nucleon over roughly 240 nucleons, of order 200 MeV) and light fusion both release energy; compare total binding, not nucleon counts.

■ **Checked example**

U-238 alpha decay: Q=(238.0508−234.0436−4.0026)×931.5≈4.28 MeV, carried mostly by the alpha by momentum conservation. D+T fusion releases ≈17.6 MeV. A sample with T½=2 h and 6×10^{12} atoms has A≈5.8×10^8 Bq. Fission of a heavy nucleus releases of order (8.5−7.6)×240≈200 MeV.

■ **Common error**

Mixing atomic and nuclear masses without tracking electrons, or reporting a positive Q for a reaction that needs energy input. Balance charge and nucleon number first, then check the sign of Q.

MM.1 · Coordinate operators, separated modes and stable updates

- Choose coordinates by symmetry before writing operators. For a radial function in spherical coordinates, $\nabla^2 f = f'' + (2/r)f'$, and the divergence of a radial field $A = A_r \hat{r}$ is $(1/r^2)d(r^2 A_r)/dr$. Check: $A = k\hat{r}/r^2$ has divergence $(1/r^2)dk/dr = 0$ away from the origin, and $f = 1/r$ satisfies $\nabla^2 f = f'' + 2f'/r = 2/r^3 - 2/r^3 = 0$ away from the origin. Cartesian forms apply only to Cartesian dependences.
- Separation of variables turns a boundary-value problem into ordinary modes. For a string fixed at both ends, write $u = X(x)T(t)$; the wave equation gives $X''/X = (1/c^2)T''/T = -k^2$, the fixed ends force $X = \sin(n\pi x/L)$, and the allowed frequencies are $\omega_n = n\pi c/L$, $f_n = nc/(2L)$. Each mode must satisfy the boundary conditions before any sum over modes.
- An explicit Euler update for $y' = -\lambda y$ is $y_{n+1} = (1 - \lambda h)y_n$, bounded when $|1 - \lambda h| \leq 1$, i.e. $0 < h \leq 2/\lambda$ for $\lambda > 0$; asymptotic decay needs $0 < h < 2/\lambda$. With $\lambda = 10$ /s, h must not exceed 0.2 s; h = 0.25 s multiplies by -1.5 each step and diverges with alternating sign. Stability is a property of the recurrence, not of the true solution.
- Check a numerical answer three ways before trusting it: units of every term (λh must be dimensionless), convergence under refinement (halve h and compare; explicit Euler error is first order), and a limiting case with a known exact answer. For $y' = -4y$ with $y0 = 8$ and $h = 0.2$, five steps give $8 \times 0.2^5 = 0.00256$ versus exact $8e^{\wedge}(-4) \approx 0.1465$: stable but inaccurate, so refine.

■ Checked example

Radial check: $f = 1/r$ gives $f'' + 2f'/r = 0$ away from the origin. A string with L = 0.5 m and c = 100 m/s has $f_1 = c/(2L) = 100$ Hz. Euler stability for $\lambda = 10$ /s needs $h \leq 0.2$ s; refining $y' = -4y$ with $y(0) = 8$ at t = 1 s, from h = 0.2 to 0.1 to 0.05 gives 0.00256, 0.0483, 0.0922, approaching the exact 0.1465.

■ Common error

Applying the Cartesian Laplacian to a radial function, summing modes that violate the boundary conditions, or calling a bounded-looking run converged without refining the step. Match operator, boundaries and stability condition to the actual problem.

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