

C.8 · Trigonometric phase and parametric curves

Original classroom teaching; not an official form or score predictor

1. Scope and prerequisites

Scope and prerequisites

Undergraduate GRE preparation. Local objectives within the reviewed ETS scope; this is original teaching, not an official test or score predictor.

2. Scope and prerequisites

Scope and prerequisites

Prerequisites: Radian trigonometry, derivatives and curve parametrisation.

3. Scope and prerequisites

Scope and prerequisites

Distinguish amplitude, angular frequency, phase angle and horizontal shift Eliminate a parameter while retaining its domain and tracing direction Use parametric derivatives without assuming a vertical tangent is stationary

4. Scope and prerequisites

Scope and prerequisites

phase angle : The angle offset inside a periodic function argument, defined modulo a full period.

5. Scope and prerequisites

Scope and prerequisites

parametric curve : A curve whose coordinates are specified as functions of a shared parameter.

6. Choose and justify a method

Choose and justify a method

For $y=A \cos(\omega x-\phi)$ with $A>0$ and $\omega>0$, amplitude is A , period is $2\pi/\omega$, and horizontal shift is ϕ/ω . The phase angle ϕ is measured inside the cosine argument; it is not itself the horizontal shift unless $\omega=1$. Start with peak-to-peak spacing for the period and midline-to-peak distance for amplitude. To find phase, substitute a known point and check the direction of motion there. Equivalent phases differ by 2π .

7. Choose and justify a method

Choose and justify a method

For $y = -2 \cos(3x)$, rewrite the model with positive amplitude as $2 \cos(3x - \pi)$. Its amplitude is 2, period $2\pi/3$, and phase π ; the equivalent right shift is $\pi/3$. Peaks occur where $3x - \pi$ is a multiple of 2π . A graph point at $x=0$ and $y=0$ alone cannot determine phase uniquely: slopes or another point distinguish the possible angles. Units and angle conventions matter; ordinary calculus trigonometric derivatives use radians.

8. Choose and justify a method

Choose and justify a method

A parametric curve specifies $x=x(t)$, $y=y(t)$. Eliminating t describes a point set but can lose restrictions and direction. For $x=\cos^3 t$, $y=\sin^3 t$, real cube roots give $|x|^{2/3}+|y|^{2/3}=1$, an astroid with cusps on the axes. As t runs from 0 to 2π it starts at $(1,0)$, passes $(0,1)$ at $\pi/2$ and travels counterclockwise. Restricting t to $[0,\pi/2]$ gives only the first-quadrant arc, not the entire implicit locus.

9. Choose and justify a method

Choose and justify a method

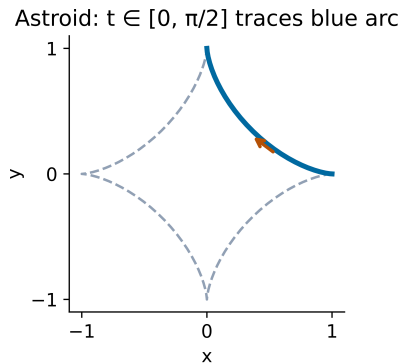
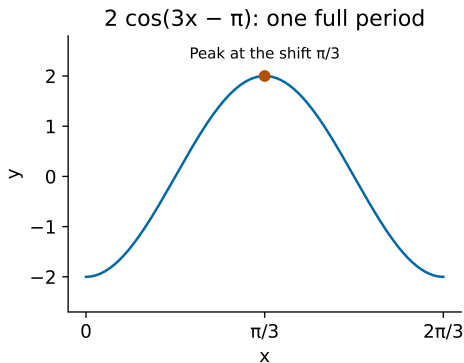
When $dx/dt \neq 0$, $dy/dx = (dy/dt)/(dx/dt)$. A horizontal tangent generally requires $dy/dt = 0$ and $dx/dt \neq 0$; a vertical tangent generally reverses those conditions. If both vanish, inspect a limit or the local expansion rather than taking $0/0$ as a slope. For $x=t^2, y=t^3$ at $t=0$, the quotient for $t \neq 0$ is $3t/2$ and tends to zero: the cusp has a horizontal tangent. A zero parameter velocity is not by itself a local maximum or minimum of y as a function of x .

10. Worked reasoning

Worked reasoning

For $y = -2 \cos(3x)$, minima include $x=0$ and maxima include $x=\pi/3$. The phase representation $2 \cos(3x-\pi)$ gives the same values. For $x=2 \cos t, y=\sin t$, elimination gives $x^2/4 + y^2 = 1$; at $t=\pi/4$, $dx/dt = -\sqrt{2}$ and $dy/dt = \sqrt{2}/2$, hence slope $-1/2$. The full parameter interval $[0, 2\pi]$ traces the ellipse once counterclockwise.

Figure 11: Worked reasoning



Check the labelled values and conditions.

12. Conditions and counterexamples

Conditions and counterexamples

Phase angle and horizontal shift differ by the frequency factor. An implicit equation can add untraced parts of a curve. If both parameter derivatives vanish, use local reasoning instead of labelling every such point an extremum.

13. Try independently

Try independently

For $y = -3 \cos(2x)$, state amplitude, period, a positive-amplitude phase representation and the corresponding horizontal shift. Parameterise the ellipse $x^2/9 + y^2/4 = 1$ counterclockwise.

14. Reasoning and check

Reasoning and check

Amplitude is three and period $2\pi/2 = \pi$. Write $y = 3\cos(2x - \pi)$, with phase π and rightward shift $\pi/2$. Phase and shift are not the same. One parametrisation is $x = 3\cos t, y = 2\sin t$, $0 \leq t \leq 2\pi$, which begins at $(3, 0)$ and initially moves upward. Eliminating t checks the ellipse equation.

15. Try independently

Try independently

For $x = t^2, y = t^3$ on $-1 \leq t \leq 1$, determine the traced locus and tangent at $t=0$. Does vanishing parameter velocity prove there is no tangent? Does the implicit equation alone retain the whole parameter restriction?

16. Reasoning and check

Reasoning and check

Elimination gives $y^2 = x^3$, with $0 \leq x \leq 1$ and both signs of y . For t nonzero, $dy/dx = 3t/2 \rightarrow 0$. The secant slope from the origin is $y/x = t \rightarrow 0$ too, so the cusp has horizontal tangent. Both parameter derivatives vanish at zero, making the raw quotient $0/0$ inconclusive, not proving absence of a tangent. The unrestricted implicit equation also has points with $x > 1$; the interval restriction must be carried separately.

Check yourself

Reconstruct the method without looking back

For $x = t^2, y = t^3$ on $-1 \leq t \leq 1$, determine the traced locus and tangent at $t=0$. Does vanishing parameter velocity prove there is no tangent? Does the implicit equation alone retain the whole parameter restriction?