

T.2 · Discrete mathematics, probability and numerical methods

Original classroom teaching; not an official form or score predictor

1. Scope and prerequisites

Scope and prerequisites

Undergraduate GRE preparation. Local objectives within the reviewed ETS scope; this is original teaching, not an official test or score predictor.

2. Scope and prerequisites

Scope and prerequisites

Prerequisites: Finite counting, elementary probability, recurrences and derivatives.

3. Scope and prerequisites

Scope and prerequisites

Use counting, recurrences, graph reasoning and logic Calculate probabilities and distribution properties Apply numerical approximation and assess error

4. Scope and prerequisites

Scope and prerequisites

recurrence : A rule linking a sequence term to earlier terms.

5. Scope and prerequisites

Scope and prerequisites

mutually exclusive : Events that cannot happen together.

6. Choose and justify a method

Choose and justify a method

Choose the counting model first: ordered selections of r distinct objects use $n(n-1)\cdots(n-r+1)$, while unordered subsets use $C(n,r)$. A complete simple graph on n vertices has one edge per unordered vertex pair, giving $n(n-1)/2$ edges; loops and multiple edges would change the model. For n lines in general position in a plane, the k th line crosses the prior $k-1$ lines in distinct points and adds k regions. The total is $1+n(n+1)/2$; parallels or triple concurrence invalidate that count.

7. Choose and justify a method

Choose and justify a method

Use complements to count at least one occurrence, and condition on the actual remaining population after a draw without replacement. For r iid outcomes chosen from n equally likely values, the probability that all are distinct is $n(n-1)\cdots(n-r+1)/n^r$ when $r \leq n$. Its complement counts repeated outcomes. For a binomial count with N independent trials and fixed success probability p , mean is Np and variance $Np(1-p)$. Identical probabilities alone do not establish independence.

8. Choose and justify a method

Choose and justify a method

A recurrence describes later values from earlier ones and needs enough initial data to determine a sequence. Separate the index from the value: $a_n = 2a_{(n-1)}$ with $a_0 = 3$ gives $a_n = 3 \cdot 2^n$. Graph and algorithm arguments often establish a recurrence by identifying what a new vertex or step adds. A closed formula should satisfy both the recurrence and its initial conditions; fitting a few observed terms does not prove it for every index.

9. Choose and justify a method

Choose and justify a method

Numerical approximation needs an error argument. Bisection preserves a sign-changing bracket for a continuous function and halves its width at each step; a zero may be absent if continuity fails. Newton's update is $x_{\text{new}} = x - f(x)/f'(x)$, requiring a nonzero derivative at the current point; convergence is not automatic from every starting value. An approximation's residual and its error in x are different. State the method's assumptions and a stopping criterion, rather than treating extra displayed decimals as accuracy.

10. Worked reasoning

Worked reasoning

For $f(x)=x^2-2$ and $x_0=1$, Newton's step gives $1-(1-2)/2=1.5$. The next value is $1.5-0.25/3=1.4167$. These approximate $\sqrt{2}$, but $f'(0)=0$ makes zero an invalid starting point.

Figure 11: Worked reasoning

Four labelled independent draws, five equally likely symbols

Event	Count model	Count
ordered strings	$5 \times 5 \times 5 \times 5$	625
all distinct	$5 \times 4 \times 3 \times 2$	120
at least one repeat	$625 - 120$	505

Check the labelled values and conditions.

12. Conditions and counterexamples

Conditions and counterexamples

Events with positive probabilities cannot be both independent and mutually exclusive.

13. Try independently

Try independently

Four labelled draws are independent and uniform from five symbols, with replacement. Find the probability that at least two draws agree. Compare the number of ordered and unordered selections of four distinct symbols.

14. Reasoning and check

Reasoning and check

There are 5^4 equally likely ordered draw strings. The no-repeat strings number $5 \cdot 4 \cdot 3 \cdot 2 = 120$. $P(\text{repeat}) = 1 - \frac{5 \cdot 4 \cdot 3 \cdot 2}{5^4} = \frac{101}{125}$. Ordered distinct selections number 120; unordered subsets number $\binom{5}{4} = 5$. Each subset occurs in $4! = 24$ orders. Dividing by $4!$ is appropriate only for the unordered count, not the draw probability denominator.

15. Try independently

Try independently

Apply bisection to $f(x) = x^3 - 2$ on $[1, 2]$. Give the first two retained brackets and a bound for midpoint error after ten bisections. Would a sign change alone justify bisection for $1/x$ on $[-1, 1]$?

16. Reasoning and check

Reasoning and check

The polynomial is continuous; $f(1) < 0 < f(2)$. The first midpoint is 1.5 with positive value, retaining $[1, 1.5]$. The next is 1.25 with negative value, retaining $[1.25, 1.5]$. After ten halvings, width is 2^{-10} and the midpoint of that retained bracket differs from the bracketed root by at most $2^{-11} = 1/2048$. The error bound uses the bracket, not a small residual alone. For $1/x$, zero is a singularity and there is no root. Continuity on the bracket fails; opposite endpoint signs do not justify the method.

Check yourself

Reconstruct the method without looking back

Apply bisection to $f(x) = x^3 - 2$ on $[1, 2]$. Give the first two retained brackets and a bound for midpoint error after ten bisections. Would a sign change alone justify bisection for $1/x$ on $[-1, 1]$?