

Solutions

2.1 C. The least positive k with $8k$ divisible by 20 is 5; equivalently $20/\gcd(8,20)=5$. Two, four and eight multiples of eight are not zero modulo twenty; twenty works but is not the least positive count.

2.2 B. Fibres are kernel cosets of size 5, so the image has $30/5=6$ elements regardless of the codomain size. Five is the kernel size; ten, twenty-five and thirty do not equal the number of five-element fibres in thirty elements.

2.3 $H = \{0, 6, 12\}$ has size three. The six cosets are $r + H = \{r, r + 6, r + 12\}$ for $r = 0, 1, 2, 3, 4, 5$. They partition all 18 residues. The element 6 has order three, while the quotient has order six. Reduction modulo six is onto with kernel H , so $(\mathbb{Z}/18\mathbb{Z})/H \cong \mathbb{Z}/6\mathbb{Z}$. Normality holds because the original group is abelian.

3.1 Conjugation by (123) sends (12) to (23) , which is outside H . Thus H is not normal. Left cosets still partition S_3 , but multiplication of those cosets is not well defined independently of representatives. The disjoint cycles in $(123)(45)$ have lengths three and two, so its order is $\text{lcm}(3, 2) = 6$. Element order and subgroup index are different quantities.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.