

Solutions

2.1 A. The initial condition fixes $C=3$. The second exponential has wrong initial value and rate; the linear expression has derivative two but twice its value is not constantly two.

2.2 B. The factor t supplies a second independent solution. Two multiples of the same exponential combine into one solution. Sine and cosine satisfy the characteristic equation $r^2 + 1 = 0$, not a repeated root minus one.

2.3 The characteristic polynomial is $(r - 2)^2$. Independent solutions are e^{2t} and te^{2t} , hence $y = (C_1 + C_2t)e^{2t}$. The initial value gives $C_1 = 1$; the derivative at zero gives $2C_1 + C_2 = 0$, so $C_2 = -2$.

$$y(t) = (1 - 2t)e^{2t}, \quad y'(t) = -4te^{2t}, \quad y''(t) = (-4 - 8t)e^{2t}.$$

Substitution gives $y'' - 4y' + 4y = 0$, and the two initial values hold. Two copies of e^{2t} would not span the solution space.

3.1 One solution is identically zero. For any $a \geq 0$, set $y_a(t) = 0$ for $0 \leq t \leq a$ and $y_a(t) = (t - a)^2$ for $t > a$. The derivative is zero on the first interval and $2(t - a)$ on the second; both sides have derivative zero at a . Therefore y_a is differentiable and satisfies the equation and initial value. The right side is continuous but is not locally Lipschitz in y at zero: $|2\sqrt{h} - 0|/|h| = 2/\sqrt{h}$ is unbounded. Existence does not imply uniqueness, and division by $\sqrt{y}$ would lose the waiting-time solutions.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.