

7.1 Complex analysis and residues

Name: _____ Class: _____ Date: _____ Total: 10 marks

KEY WORDS analytic • residue

Objective

Test complex differentiability; Use contour integrals and residues; Evaluate contour integrals with simple and higher-order pole residues.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

1 Key knowledge and worked examples

analytic : Complex differentiable throughout a neighbourhood.

residue : Coefficient of $(z-a)^{-1}$ in a Laurent series.

Write $f(z)=u(x,y)+iv(x,y)$. Complex differentiability imposes $u_x=v_y$ and $u_y=-v_x$; with continuous first partials locally, the Cauchy–Riemann equations establish analyticity there. They differ from real differentiability of a two-coordinate map. The conjugate function $x-iy$ fails these equations on every open neighbourhood, although real partial derivatives exist. State the region being checked, not only a convenient point.

For $f(z)=\text{conjugate}(z)$, $u=x$ and $v=-y$. Then $u_x=1$ but $v_y=-1$, so f is not complex differentiable. For $1/(z-2)$, a positively oriented circle $|z-2|=1$ encloses a simple pole of residue 1, hence the integral is $2\pi i$.

The residue is the coefficient of $(z - 1)^{-1}$

Function	Pole order	Residue at 1
$3/(z - 1)$	1	3
$e^z/(z - 1)^2$	2	e
$e^{2z}/(z - 1)^3$	3	$2e^2$

Original course illustration; its values belong to the worked example, not the later practice.

2 Practice

2.1 The residue of $3/(z-1)$ at $z=1$ is: [1]

- A 1
- B 3
- C 0
- D $2\pi i$
- E $6\pi i$

2.2 A positively oriented circle enclosing only that pole gives integral: [1]

- A 0
- B $3\pi i$
- C $6\pi i$
- D 3
- E $-6\pi i$

2.3 Use the Cauchy–Riemann equations to compare z^2 and $\bar{z}$. Find the residue of e^z/z^2 at zero and its integral around the counterclockwise unit circle. [4]

3 Exam-style questions

3.1 Integrate $1/[z(z-2)]$ around $|z|=1$, first counterclockwise and then clockwise. Can the same residue-theorem argument be used on $|z|=2$? [4]
