

Solutions

2.1 B. The coefficient of the simple-pole term is 3. One is the pole location, not its coefficient. Zero loses the pole term; $2\pi i$ and $6\pi i$ are integral-like quantities, not the Laurent coefficient three.

2.2 C. Multiply residue 3 by $2\pi i$. Zero omits the pole; $3\pi i$ misses the factor two; three is the residue; negative $6\pi i$ reverses orientation.

2.3 For z^2 , $u = x^2 - y^2$ and $v = 2xy$: $u_x = 2x = v_y$ and $u_y = -2y = -v_x$. The partials are continuous everywhere, so the function is analytic everywhere. For $\bar{z}$, $u = x$, $v = -y$ and $u_x = 1 \neq -1 = v_y$; it is nowhere complex differentiable. Since $e^z = 1 + z + z^2/2 + \cdots$, the coefficient of z^{-1} in e^z/z^2 is 1. Therefore the residue is 1 and the integral is $2\pi i$. The double pole does not force a zero residue.

3.1 Only zero is inside the unit circle. Its residue is $\lim_{z \rightarrow 0} 1/(z - 2) = -1/2$. Thus the counterclockwise integral is $-\pi i$ and the clockwise integral is πi . The pole at 2 is outside and contributes nothing. The circle $|z| = 2$ passes through a pole. The ordinary contour integral is not defined by this formula; the theorem requires analyticity on the contour. An indentation or principal-value prescription would be a different, explicitly specified problem.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.