

# 4.1 Abstract algebra and number theory

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 10 marks****KEY WORDS** homomorphism • field

## Objective

Use groups, subgroups, homomorphisms and quotient structures; Distinguish rings, integral domains and fields; Apply divisibility, congruences and elementary number theory.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

## 1 Key knowledge and worked examples

**homomorphism** : A map preserving the relevant operation.

**field** : A commutative ring where each nonzero element has an inverse.

A group needs closure, associativity, identity and inverses. Commutativity is an additional condition.

Modulo 8, 3 has inverse 3 because  $3 \cdot 3 = 9 \equiv 1$ . But 2 has no inverse because  $\gcd(2, 8) = 2$ . Modulo a prime, every nonzero residue has an inverse; this gives a finite field.

### Same addition rule; different invertibility

| Ring  | Multiplicative units | Check                 |
|-------|----------------------|-----------------------|
| mod 5 | 1, 2, 3, 4           | every nonzero residue |
| mod 6 | 1, 5                 | 2 times 3 is zero     |

*Original course illustration; its values belong to the worked example, not the later practice.*

## 2 Practice

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**2.1** The integers under addition form: [1]

- **A** a group
  - **B** a field under integer division
  - **C** a set without identity
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**2.2** Which has an inverse modulo 9? [1]

- **A** 3
  - **B** 6
  - **C** 2
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**2.3** Compare the additive groups and multiplicative structures of  $\mathbb{Z}/5\mathbb{Z}$  and  $\mathbb{Z}/6\mathbb{Z}$ . List all multiplicative units in each. Explain why removing zero does not always leave a multiplicative group. [4]

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### 3 Exam-style questions

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**3.1** The reduction map  $\phi : \mathbb{Z} \rightarrow \mathbb{Z}/6\mathbb{Z}$  is given by  $\phi(n) = [n]$ . Determine its kernel, image and quotient. Solve  $2x \equiv 2 \pmod{6}$  completely and explain why cancelling 2 modulo 6 fails. [4]

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