

23.1 Trigonometric phase and parametric curves

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS phase angle • parametric curve

Objective

Distinguish amplitude, angular frequency, phase angle and horizontal shift; Eliminate a parameter while retaining its domain and tracing direction; Use parametric derivatives without assuming a vertical tangent is stationary.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

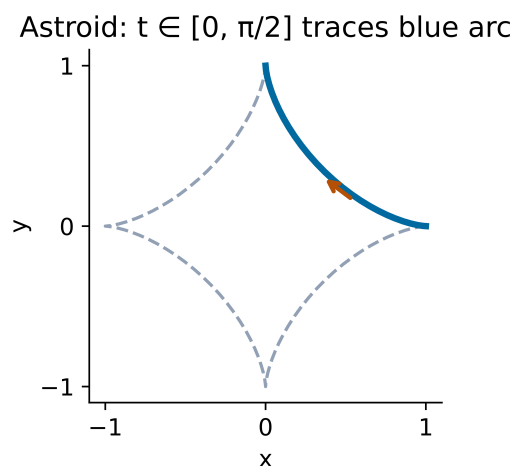
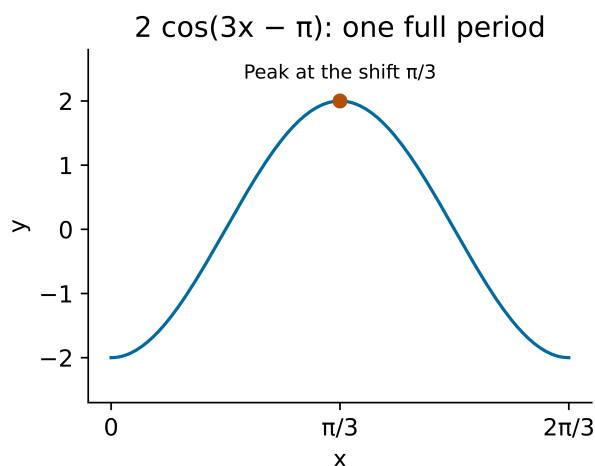
1 Key knowledge and worked examples

phase angle : The angle offset inside a periodic function argument, defined modulo a full period.

parametric curve : A curve whose coordinates are specified as functions of a shared parameter.

For $y = A \cos(\omega x - \varphi)$ with $A > 0$ and $\omega > 0$, amplitude is A , period is $2\pi/\omega$, and horizontal shift is φ/ω . The phase angle φ is measured inside the cosine argument; it is not itself the horizontal shift unless $\omega = 1$. Start with peak-to-peak spacing for the period and midline-to-peak distance for amplitude. To find phase, substitute a known point and check the direction of motion there. Equivalent phases differ by 2π .

For $y = -2 \cos(3x)$, minima include $x = 0$ and maxima include $x = \pi/3$. The phase representation $2 \cos(3x - \pi)$ gives the same values. For $x = 2 \cos t, y = \sin t$, elimination gives $x^2/4 + y^2 = 1$; at $t = \pi/4$, $dx/dt = -\sqrt{2}$ and $dy/dt = \sqrt{2}/2$, hence slope $-1/2$. The full parameter interval $[0, 2\pi]$ traces the ellipse once counterclockwise.



Original course illustration; its values belong to the worked example, not the later practice.

2 Practice

2.1 For $y = 2 \cos(3x - \pi)$, what is the rightward horizontal shift?

[1]

- **A** π
- **B** 3π
- **C** $\pi/3$
- **D** $2\pi/3$
- **E** $1/3$

2.2 For $x = \cos^3 t, y = \sin^3 t$ with t in $[0, \pi/2]$, which set is traced?

[1]

- **A** The whole unit circle
- **B** The whole astroid
- **C** Only the first-quadrant astroid arc
- **D** Only the line $x + y = 1$
- **E** A circle of radius 3

2.3 For $y = -3 \cos(2x)$, state amplitude, period, a positive-amplitude phase representation and the corresponding horizontal shift. Parameterise the ellipse $x^2/9 + y^2/4 = 1$ counterclockwise. [4]

3 Exam-style questions

3.1 For $x = t^2, y = t^3$ on $-1 \leq t \leq 1$, determine the traced locus and tangent at $t=0$. Does vanishing parameter velocity prove there is no tangent? Does the implicit equation alone retain the whole parameter restriction? [4]
