

22.1 Similarity, scaling and conic distance loci

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS similarity • focus

Objective

Match triangle vertices by equal angles before forming a side ratio; Apply length, area and volume scale factors to geometric changes; Identify conic distance conditions and retain a signed hyperbola branch.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

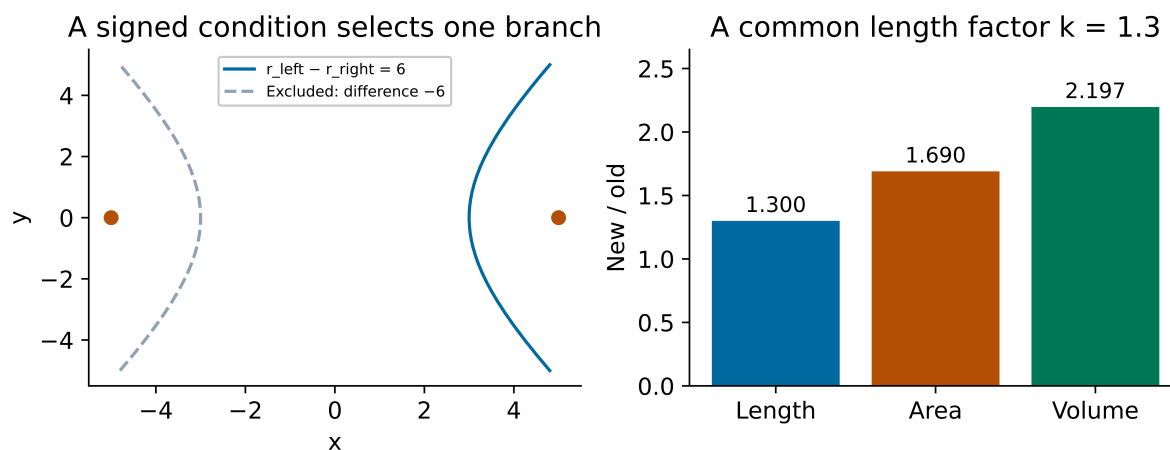
1 Key knowledge and worked examples

similarity : Equality of corresponding angles and a common ratio of corresponding lengths.

focus : A fixed point used in a conic distance definition.

Triangles are similar when their corresponding angles agree; their corresponding side ratios are then equal. Write the vertex correspondence explicitly. If triangle APQ has angle A equal to angle A of ABC and angle P equal to angle C, its ordered correspondence is $A \leftrightarrow A$, $P \leftrightarrow C$, $Q \leftrightarrow B$. Thus $AP/AC = AQ/AB = PQ/CB$. The side AP lies on AB but corresponds to AC; matching by where a side lies or by the letter P can give the wrong ratio.

Let $AB=10$, $AC=15$ and $BC=20$. A point P on AB has $AP=6$, and Q on AC makes angle APQ equal to angle ACB. Then $AP/AC=6/15=2/5$, so $PQ=8$ and $AQ=4$. Separately, foci at $(-5,0)$, $(5,0)$ and distance-to-left minus distance-to-right equal to 6 give $a=3$, $b^2=25-9=16$: $x^2/9 - y^2/16 = 1$ with $x \geq 3$. The point $(-3,0)$ satisfies the squared equation but fails the signed condition.



Original course illustration; its values belong to the worked example, not the later practice.

2 Practice

2.1 Correspondence is $A \leftrightarrow A$, $P \leftrightarrow C$, $Q \leftrightarrow B$. If $AP=6$, $AC=15$ and $CB=20$, what is PQ ?^[1]

- A 4
- B 6
- C 8
- D 12
- E 50

2.2 A circle radius grows by 30%. What is the area increase?

^[1]

- A 30%
- B 60%
- C 69%
- D 90%
- E 169%

2.3 Similar solids have surface-area ratio 9:25. Find their corresponding length and volume ratios. Explain why matching vertices matters before forming a triangle ratio.[4]

3 Exam-style questions

3.1 For foci $A = (-5, 0)$ and $B = (5, 0)$, find the locus $|PA| - |PB| = 6$. Derive its standard equation, choose its branch and check the vertex in the unsquared relation. What happens if the requested difference is 12? [4]
