

Solutions

2.1 B. Treat y as constant. A differentiates the wrong factor; C drops the fixed factor y . At $y=0$ every derivative value is zero, consistently with $2xy$.

2.2 B. The Jacobian of the polar transformation is r . A omits the local scale r ; C is the radial power for a three-dimensional spherical volume, with an additional angular factor, not a planar area element.

2.3 The polynomial is differentiable. Normalise the direction: $u = (3/5, 4/5)$. The gradient is $(2x + y, x + 4y)$, hence $(1, -3)$ at the point.

$$D_u f(1, -1) = \nabla f(1, -1) \cdot u = 1 \cdot 3/5 - 3 \cdot 4/5 = -9/5.$$

Polar coordinates give $x^2 + y^2 = r^2$ and $dA = r \, dr \, d\theta$:

$$I = \int_0^{2\pi} \int_0^1 r^3 \, dr \, d\theta = \pi/2.$$

3.1 Along each axis $g = 0$, so both partials are zero. Continuity follows from $|xy| \leq (x^2 + y^2)/2$, which gives $|g(x, y)| \leq \sqrt{x^2 + y^2}/2 \rightarrow 0$. If differentiable, its linear derivative would be zero. Along $x = y = t \neq 0$,

$$\frac{|g(t, t)|}{\sqrt{t^2 + t^2}} = \frac{1}{2}.$$

The required remainder ratio does not tend to zero. Thus continuity and existing partial derivatives do not establish differentiability.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.