

Solutions

2.1 A. The input restriction $x \geq 2$ forces $x - 2 \geq 0$, so select the positive square-root branch. The negative branch violates $x \geq 2$; square root of $y - 5$ has wrong shift. A reciprocal is not an inverse function, and the last quadratic does not undo the original square.

2.2 C. The orbit is 3, $-1/2$, $2/3$, then 3 again. Eight is congruent to two modulo three. Three is the zero-mod-three iterate; minus one-half is the one-mod-three iterate. One over 256 treats composition as multiplication, and nine squares the input instead of iterating the map.

2.3 The image is $[2, \infty)$. Solving $y = (x - 1)^2 + 2$ with $x \geq 1$ selects $x = 1 + \sqrt{y - 2}$. Thus $f^{-1} : [2, \infty) \rightarrow [1, \infty)$ has that formula. For allowed x , $1 + \sqrt{(x - 1)^2} = x$ because $x - 1 \geq 0$. For allowed y , $(\sqrt{y - 2})^2 + 2 = y$. The negative square-root branch belongs to a different restriction, not a second inverse on this domain.

3.1 The first stage requires $x \neq 1$. The second requires $h(x) \neq 1$, which excludes $x = 0$. For $x \neq 0, 1$, $h^2(x) = (x - 1)/x$. This never equals one, so the third stage adds no exclusion. Then $h^3(x) = x$ on $\mathbb{R} \setminus \{0, 1\}$. The identity formula is defined more widely, but adding 0 or 1 defines an extension, not the original composite. Each intermediate input must be permitted before cancellation.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.