

Solutions

2.1 D. The coefficient is $2^3/3!=8/6=4/3$; the factorial cannot be omitted. One-sixth is the coefficient for e^x ; one-third and two-thirds omit some powers of two; eight omits division by factorial three.

2.2 D. The radius is 1 and both endpoints converge absolutely by comparison with $\sum 1/n^2$. Each open or half-open interval wrongly discards at least one absolutely convergent endpoint; all of $\mathbb{R}$ includes inputs with magnitude greater than one where terms fail to tend to zero.

2.3 The ratio test gives radius one. At 1 the original harmonic series diverges; at -1 the alternating series converges. Thus the original interval is $[-1, 1)$. The derivative is $\sum_{n=1}^{\infty} x^{n-1} = 1/(1-x)$ only for $|x| < 1$; its terms do not tend to zero at either endpoint. $P_2(0.1) = 1+0.1+0.1^2/2 = 1.105$. On $[0, 0.1]$, the third derivative is $e^x < 2$. The Lagrange bound gives $|R_2| \leq 2(0.1)^3/6 = 1/3000$. The bound is deliberately conservative, not a claim of exact error.

3.1 Taylor's theorem near zero gives $\cos x = 1 - x^2/2 + x^4/24 + O(x^6)$. Subtraction cancels the constant and quadratic terms, leaving $x^4/24 + O(x^6)$. After division the limit is $1/24$. A truncation with remainder merely $o(x^2)$ gives no control after division by x^4 ; retain and bound the first surviving order.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.