

Solutions

2.1 C. Bolzano–Weierstrass gives a convergent subsequence. The sequence $(-1)^n$ rules out the other necessary claims. The bounded sequence $(-1)^n$ is neither convergent, monotone nor Cauchy and does not tend to zero. Its even subsequence does converge.

2.2 C. The supremum of the absolute value is $1/n$, independent of x , which tends to zero. A, D and E fail because the uniform error bound includes every real x , including zero and noninteger inputs. B fails because the same bound $1/n$ tends to zero independently of x .

2.3 On $[0, 1]$, the limit is zero for $x < 1$ and one at 1. This limit is discontinuous, so continuous f_n cannot converge uniformly. More directly the supremum error on $[0, 1)$ is one. On $[0, a]$ the supremum is $a^n \rightarrow 0$, so convergence is uniform to zero. The p -series with $p=2$ converges absolutely. The alternating harmonic series converges by decreasing terms tending to zero, but the absolute series diverges; its convergence is conditional.

3.1 $\sup_{x \in \mathbb{R}} |g_n(x)| = 1/n \rightarrow 0$, so the functions converge uniformly to zero. But $g'_n(x) = \cos(nx)$, and at $x = 0$ these derivatives are all 1. The derivative of the zero limit is 0. Thus uniform convergence of the functions alone does not justify differentiating a limit; additional derivative-convergence hypotheses are needed.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.