

23.1 Trigonometric phase and parametric curves

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS phase angle • parametric curve

Objective

Distinguish amplitude, angular frequency, phase angle and horizontal shift; Eliminate a parameter while retaining its domain and tracing direction; Use parametric derivatives without assuming a vertical tangent is stationary.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

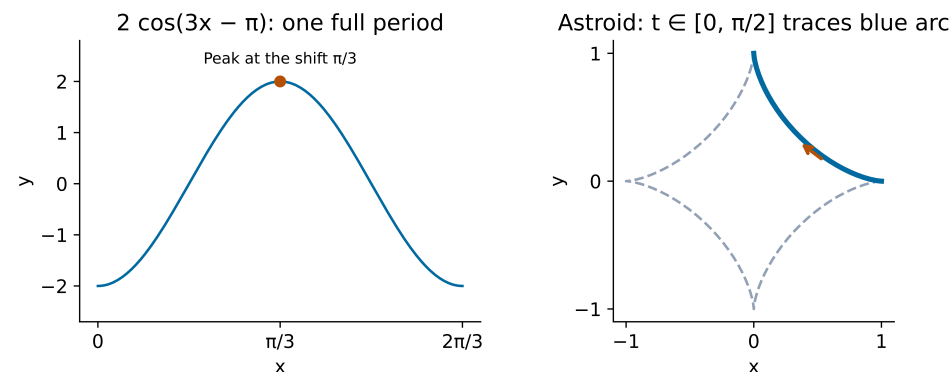
1 Key knowledge and worked examples

phase angle : The angle offset inside a periodic function argument, defined modulo a full period.

parametric curve : A curve whose coordinates are specified as functions of a shared parameter.

For $y = A \cos(\omega x - \varphi)$ with $A > 0$ and $\omega > 0$, amplitude is A , period is $2\pi/\omega$, and horizontal shift is φ/ω . The phase angle φ is measured inside the cosine argument; it is not itself the horizontal shift unless $\omega = 1$. Start with peak-to-peak spacing for the period and midline-to-peak distance for amplitude. To find phase, substitute a known point and check the direction of motion there. Equivalent phases differ by 2π .

For $y = -2 \cos(3x)$, minima include $x = 0$ and maxima include $x = \pi/3$. The phase representation $2 \cos(3x - \pi)$ gives the same values. For $x = 2 \cos t, y = \sin t$, elimination gives $x^2/4 + y^2 = 1$; at $t = \pi/4$, $dx/dt = -\sqrt{2}$ and $dy/dt = \sqrt{2}/2$, hence slope $-1/2$. The full parameter interval $[0, 2\pi]$ traces the ellipse once counterclockwise.



Original course illustration; its values belong to the worked example, not the later practice.

2 Practice

2.1 For $y = 2 \cos(3x - \pi)$, what is the rightward horizontal shift? [1]

- A π
- B 3π
- C $\pi/3$
- D $2\pi/3$
- E $1/3$

2.2 For $x = \cos^3 t, y = \sin^3 t$ with t in $[0, \pi/2]$, which set is traced? [1]

- A The whole unit circle
- B The whole astroid
- C Only the first-quadrant astroid arc
- D Only the line $x + y = 1$
- E A circle of radius 3

2.3 For $y = -3\cos(2x)$, state amplitude, period, a positive-amplitude phase representation and the corresponding horizontal shift. Parameterise the ellipse $x^2/9 + y^2/4 = 1$ counterclockwise. [4]

3 Exam-style questions

3.1 For $x = t^2, y = t^3$ on $-1 \leq t \leq 1$, determine the traced locus and tangent at $t=0$. Does vanishing parameter velocity prove there is no tangent? Does the implicit equation alone retain the whole parameter restriction? [4]

Solutions

2.1 C. Factor the argument as $3(x - \pi/3)$. The phase π is divided by angular frequency 3. π is the phase before division; three π multiplies rather than divides; two π over three is the period, and one-third omits the phase π .

2.2 C. The parameter interval keeps both coordinates nonnegative. Elimination gives the astroid relation, but does not remove this quadrant restriction. The full curves exceed the parameter interval; the circle and line have different equations. The restricted first-quadrant arc alone matches both coordinate signs and the astroid relation.

2.3 Amplitude is three and period $2\pi/2 = \pi$. Write $y = 3\cos(2x - \pi)$, with phase π and rightward shift $\pi/2$. Phase and shift are not the same. One parametrisation is $x = 3\cos t, y = 2\sin t, 0 \leq t \leq 2\pi$, which begins at $(3, 0)$ and initially moves upward. Eliminating t checks the ellipse equation.

3.1 Elimination gives $y^2 = x^3$, with $0 \leq x \leq 1$ and both signs of y . For t nonzero, $dy/dx = 3t/2 \rightarrow 0$. The secant slope from the origin is $y/x = t \rightarrow 0$ too, so the cusp has horizontal tangent. Both parameter derivatives vanish at zero, making the raw quotient $0/0$ inconclusive, not proving absence of a tangent. The unrestricted implicit equation also has points with $x > 1$; the interval restriction must be carried separately.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.