

Exercise walk-through

26.1

Weak compositions, loop invariants and flowchart tracing

GRE · GRE Subject Test · GRE Mathematics

8. Choose and justify a method

Choose and justify a method

Trace a flowchart one executed statement at a time, recording variables in a table. Distinguish assignment from a mathematical equality test; the right side of an assignment uses the old value before replacement. Record which branch executes next and which variables reset. A new outer-loop candidate may reset an inner sum without resetting the candidate itself. For a process that adds 1,3,5,... to S from zero, after m additions $S=m^2$ and the next addend is $2m+1$.

9. Choose and justify a method

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A loop invariant is true before and after each iteration. The square-sum invariant holds initially at $m=0$ and is preserved because $m^2 + (2m+1) = (m+1)^2$. An invariant alone does not prove termination. To test whether an integer $N \geq 0$ is square by adding odd numbers until $S \geq N$, S grows without bound; equivalently m increases and must reach a bound such as N . At exit, equality means square and overshoot means nonsquare. A printed value must be reached on an actual execution path; a plausible numerical pattern alone is not a trace.

16. Reasoning and check

Reasoning and check

After m iterations, $S = m^2$ and $k = 2m + 1$. The trace pairs (S, k) are $(0, 1), (1, 3), (4, 5), (9, 7), (16, 9)$. Thus 15 is rejected by overshoot and 16 accepted. The invariant holds initially and is preserved since $m^2 + (2m + 1) = (m + 1)^2$. The sum grows without bound, so it eventually reaches or exceeds N ; for example $m = N$ suffices when N is positive. For $N = 0$ the loop is skipped and equality holds. An invariant alone would not establish termination.

Exercise 2.1

2.1 How many ways put 11 identical tokens into 4 named boxes, allowing empty boxes?^[1]

- **A** 44
- **B** 120
- **C** 244
- **D** 364
- **E** 4^{11}

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- **A** 44
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2.1 D. There are 11 stars and 3 bars. Choose the 3 bar positions among 14 positions: $C(14,3)=364$. Stars and bars gives $C(14,3)$. Forty-four, 120 and 244 do not count that arrangement; 4 to the eleventh counts distinguishable tokens and therefore a different sample space.

Exercises 2.2·2.3

2.2 Starting from $S=0$, add successive odd numbers 1,3,5,... until $S \geq 10$. What is S at exit? [1]

- **A** 9
- **B** 10
- **C** 11
- **D** 16
- **E** 25

2.3 Count nonnegative solutions of $a + b + c = 8$ with $a \geq 2$. How many also have $b, c \geq 1$? How many of the first set have at least one of b, c equal to zero? [4]

Exercises 2.2-2.3

2.2 Starting from $S=0$, add successive odd numbers $1,3,5,\dots$ until $S \geq 10$. What is S at exit? [1]

- A 9
- B 10
- C 11
- D 16
- E 25

2.2 D. The successive sums are $1,4,9,16$. The first sum at least 10 is 16, so the test overshoots rather than declaring 10 square. Nine is the previous sum; ten and eleven are not square partial sums; twenty-five is reached only if the loop continues after its condition becomes false.

2.3 Count nonnegative solutions of $a + b + c = 8$ with $a \geq 2$. How many also have $b, c \geq 1$? How many of the first set have at least one of b, c equal to zero? [4]

2.3 Put $u = a - 2$. Then $u + b + c = 6$ has $\binom{8}{3} = 28$ nonnegative solutions. Requiring b, c

Exercise 3.1

3.1 Trace this algorithm for $N=15$ and $N=16$: initialise $S=0$, $k=1$; while $S < N$, execute $S=S+k$, then $k=k+2$; report whether $S=N$. State an invariant, and justify termination for integer $N \geq 0$. [4]

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3.1 After m iterations, $S = m^2$ and $k = 2m+1$. The trace pairs (S,k) are $(0, 1), (1, 3), (4, 5)$, Thus 15 is rejected by overshoot and 16 accepted. The invariant holds initially and is preserved since $m^2 + (2m+1) = (m+1)^2$. The sum grows without bound, so it eventually reaches or exceeds N ; for example $m=N$ suffices when N is positive. For $N=0$ the loop is skipped and equality holds. An invariant alone would not establish termination.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.