

- On $X = \mathbb{R}$, which h gives a complete metric $d(x,y) = |h(x) - h(y)|$?

☐ $h(x) = \frac{x}{1 + |x|}$
☐ $h(x) = \arctan x$
☐ $h(x) = 2x + 1$
☐ $h(x) = e^x$
- On $X = \{2, 3, 4, \dots\}$ use the basis $U_k = \{n \in X : n \mid k\}$. Which listed candidates lie in the closure of $\{6\}$?

☐ 18
 ☐ 20
 ☐ 6
 ☐ 3
 ☐ 12

Tick every correct option.
- For $h(x) = x^2$, $d(x,y) = |x^2 - y^2|$ is a metric on $\mathbb{R}$.

☐ True

 ☐ False
- For $X = \{2, 3, 4, \dots\}$ and $U_k = \{n \in X : n \mid k\}$, how many elements does U_{12} have?

- Under $d(x,y) = |\arctan x - \arctan y|$ on $\mathbb{R}$, the sequence $x_n = n$ for positive integers n is a _____ sequence with no d-limit in $\mathbb{R}$.

- For any map $h : X \rightarrow \mathbb{R}$, the function $d(x,y) = |h(x) - h(y)|$ satisfies the triangle inequality, even if h is not injective.

☐ True

 ☐ False
- For injective h, knowing only that $h(X)$ is closed as a subset of itself proves that the pullback metric is complete.

☐ True

 ☐ False
- The arctangent tail proof bounds every gap for $m, n \geq N$ by $1/N$. What is the smallest positive integer N satisfying $1/N < 0.01$?

- For $d(x,y) = |e^x - e^y|$ on $\mathbb{R}$, which sequence witnesses incompleteness by approaching the missing image endpoint zero?

☐ $x_n = 0$ for every n
 ☐ $x_n = n$
☐ $x_n = -n$
☐ $x_n = (-1)^n$
- Which specified map and input domain give a complete pullback metric?

☐ $h(x) = x$ on $[0, 1)$
☐ $h(x) = \arctan x$ on $\mathbb{R}$
☐ $h(x) = x^2$ on $\mathbb{R}$
☐ $h(x) = e^x$ on $[0, \infty)$

1. $h(x) = 2x + 1$

All four maps are injective. The affine map has image $\mathbb{R}$, closed in the real line. The others have images $(-\pi/2, \pi/2)$, $(0, \infty)$ and $(-1, 1)$, each missing a real limit point.

2. **18; 6; 12**

A candidate x lies in the closure exactly when its smallest neighbourhood U_x contains 6, which means $6 \mid x$. Thus the listed members are 6, 12 and 18.

3. **False**

False. The map $x \mapsto x^2$ is not injective on $\mathbb{R}$: $d(1, -1) = 0$ even though the inputs differ. Nonnegativity, symmetry and the triangle inequality still hold.

4. **5**

$U_{12} = \{2, 3, 4, 6, 12\}$ has five elements. The divisor 1 is excluded because it does not belong to X .

5. **Cauchy**

The arctangent images have tail gaps below $1/N$ for indices at least N , so the sequence is Cauchy. Their real limit $\pi/2$ is outside the image, so no real input is its d-limit.

6. **True**

True. The ordinary real triangle inequality applies to $h(x), h(y), h(z)$. Lack of injectivity can fail separation of distinct inputs, not the triangle inequality.

7. **False**

False. Every space is closed in itself. The criterion for a real image requires closedness in $\mathbb{R}$ with ordinary distance; the arctangent image is a counterexample.

8. **101**

The strict inequality requires $N > 100$. Thus the smallest integer is 101, not 100.

9. $x_n = -n$

For $x_n = -n$, the images e^{-n} tend to zero, a missing point of $(0, \infty)$. The input sequence is d-Cauchy but has no d-limit.

10. $h(x) = e^x$ **on** $[0, \infty)$

The restricted exponential map is injective with image $[1, \infty)$, closed in the real line. The first two images miss limit points; the last map is not injective.