

30. C.12 · Related rates and removable quotient limits

Starter Quiz ·

GRE Subject Test · GRE Mathematics

Name: _____ Score: _____

1. An upright cylinder of fixed radius 2 m fills with depth rate $3 \frac{\text{m}}{\text{min}}$. Its volume rate is $k\pi$ cubic metres per minute. Find k .

2. A fixed sphere of radius 5 m contains water of depth 2 m measured from the bottom. There is no inflow, and depth falls at $0.5 \frac{\text{m}}{\text{min}}$. The outflow is $k\pi$ cubic metres per minute. Find k .

3. Evaluate $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 5x}$.

4. In a fixed tank with positive horizontal area at the current interior level, $dh/dt < 0$. The signed net volume rate dV/dt is
☐ positive
☐ zero
☐ negative
☐ undetermined
5. If f/g has a finite limit at 0, its continuous extension is differentiable at 0.
☐ True ☐ False
6. A sphere has fixed radius 3 m and water depth 1 m from its bottom. Give the square of the water-disc radius, in square metres.

7. At depth $h = R$ in a sphere, write the water volume as $k\pi R^3$. Give k .

8. For a fixed spherical tank with no inlet and constant positive volume outflow, the water depth falls at a constant speed throughout the interior.
☐ True ☐ False
9. For $x \neq 0$, let $H(x) = (x + x^2)/x$. Which value must $H(0)$ have for continuity at zero?
☐ It can be any real number
☐ 2

☐ 1

☐ 0

10. A quotient with $f(0) = g(0) = g'(0) = 0$ can still have a differentiable extension at zero.

☐ True ☐ False

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Answers ·

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1. **12**

The current area is 4π square metres. Thus $dV/dt = 4\pi(3) = 12\pi$ cubic metres per minute, so $k = 12$.

2. **8**

Area $A = \pi(2Rh - h^2) = 16\pi$. The signed volume rate is $16\pi(-0.5) = -8\pi$; no inflow means positive outflow 8π , so $k = 8$.

3. **0.2**

Both functions vanish at zero and are differentiable there. Their derivatives are 1 and 5, respectively. The nonzero denominator derivative permits the local quotient limit $1/5 = 0.2$.

4. **negative**

The chain rule gives $dV/dt = A(h) dh/dt$. A positive area times a negative depth rate is negative; this is a net rate, not necessarily the outlet flow.

5. **False**

False. With $f = x|x|$ and $g = x$, the quotient is $|x|$ away from zero. Its continuous fill is 0, but the one-sided derivatives there are -1 and 1 .

6. **5**

$a^2 = R^2 - (h - R)^2 = 9 - 4 = 5$. The disc area is 5π , while the disc radius is $\sqrt{5}$.

7. **0.6666666666666666**

$V(R) = \pi(R^3 - R^3/3) = 2\pi R^3/3$, so $k = 2/3$. This is half of the full sphere volume.

8. **False**

False. The depth rate is $-q/A(h)$, and the area changes with depth. Its fall speed is smallest at the equator and grows toward either endpoint in this ideal model.

9. **1**

The nearby expression is $1 + x$, so its limit is 1. Altering the isolated value cannot change that limit.

10. **True**

True. With $f(x) = x^3$ and $g(x) = x^2$, the quotient equals x for $x \neq 0$. Filling with 0 gives a smooth function, although the earlier nonzero-derivative test does not apply.