

- Evaluate $\int_0^1 x e^x \, dx$.

- Evaluate $\int_0^1 \int_x^1 \sin(y^2) \, dy \, dx$ to three decimal places.

- For continuous $f(x, y)$ on $0 \leq x \leq 2$, $0 \leq y \leq x/2$, which expression reverses $\int_0^2 \int_0^{x/2} f(x, y) \, dy \, dx$?

☐ $\int_0^1 \int_{2y}^2 f \, dx \, dy$
☐ $\int_0^{x/2} \int_0^2 f \, dx \, dy$
☐ $\int_0^2 \int_{2y}^2 f \, dx \, dy$
☐ $\int_0^1 \int_0^{2y} f \, dx \, dy$
- Evaluate the ordinary improper integral $\int_0^\pi \sin(6x)/\sin x \, dx$.

- In $\int_{-\infty}^\infty x^2 e^{-x^2} \, dx$ by parts, the boundary term $-\frac{1}{2} x e^{-x^2}$ vanishes at both infinities.

☐ True
 ☐ False
- For $I_n(x) = \int_1^x (\ln t)^n \, dt$, give $I_0(3)$.

- Evaluate $\int_1^\infty (1 + \ln x)/x^2 \, dx$.

- If two iterated improper integrals of a sign-changing function are both finite, they must be equal.

☐ True
 ☐ False
- For continuous f on $0 \leq x \leq 1$, $x^2 \leq y \leq x$, what is the reversed integral?

☐ $\int_0^1 \int_y^{\sqrt{y}} f(x, y) \, dx \, dy$
☐ $\int_0^1 \int_{\sqrt{y}}^y f(x, y) \, dx \, dy$
☐ $\int_0^1 \int_{y^2}^y f(x, y) \, dx \, dy$
☐ $\int_0^1 \int_0^y f(x, y) \, dx \, dy$

☐ True
 ☐ False

1.
 1

Parts gives $[xe^x]_0^1 - \int_0^1 e^x \, dx = e - (e - 1) = 1$. Equivalently $[e^x(x - 1)]_0^1 = 0 - (-1) = 1$.

2.
 0.23

The continuous integrand is on $0 \leq x \leq y \leq 1$. Reverse to $\int_0^1 \int_0^y \sin(y^2) \, dx \, dy = \int_0^1 y \sin(y^2) \, dy = (1 - \cos 1)/2 \approx 0.229849$, hence 0.230.

3.
 $\int_0^1 \int_{2y}^2 f \, dx \, dy$

The outer y range is $0 \leq y \leq 1$. At fixed y, solve $y \leq x/2$ to obtain $2y \leq x \leq 2$.

4.
 0

The endpoint limits are 6 and -6 , so the singularities are removable. Reflection $x \mapsto \pi - x$ changes the sign and preserves the interval; the integral is zero.

5.
 True

True. On each finite interval the boundary product is $-xe^{-x^2}/2$. Its limit is zero at both infinities. First establish the two half-line integrals; evenness then gives the full-line value $\sqrt{\pi}/2$.

6.
 2

The base case is $I_0(x) = x - 1$, so $I_0(3) = 2$. The reduction for $n \geq 1$ must not be used with $n = 0$.

7.
 2

On $[1, R]$, parts gives boundary difference $1 - (1 + \ln R)/R$ and remaining integral $1 - 1/R$. Their sum tends to 2.

8.
 False

False. On the positive unit square the lesson\'s function $(x^2 - y^2)/(x^2 + y^2)^2$ gives $\pi/4$ and $-\pi/4$. Absolute integrability fails near the origin.

9.
 $\int_0^1 \int_y^{\sqrt{y}} f(x, y) \, dx \, dy$

From $y \leq x$ obtain $x \geq y$; from $x^2 \leq y$ and $x \geq 0$ obtain $x \leq \sqrt{y}$. The outer range is $0 \leq y \leq 1$.

10.
 True

True. Reflection maps the interval onto itself and makes the integral equal its negative. Integrability is part of the hypothesis.