29. C.11 · Integration by parts, order reversal and symmetry

Starter Quiz ·

GRE Subject Test · GRE Mathematics Name: _____ Score: _____

1. Evaluate $\int_0^1 xe^x\,dx$.

   _____

2. Evaluate $\int_0^1 \int_x^1 \sin(y^2)\,dy\,dx$ to three decimal places.

   _____

3. For continuous $f(x,y)$ on $0 \le x \le 2$, $0 \le y \le x/2$, which expression reverses $\int_0^2 \int_0^{x/2} f(x,y)\,dy\,dx$?

   ☐ $\int_0^1 \int_{2y}^2 f\,dx\,dy$

   ☐ $\int_0^{x/2} \int_0^2 f\,dx\,dy$

   ☐ $\int_0^2 \int_{2y}^2 f\,dx\,dy$

   ☐ $\int_0^1 \int_0^{2y} f\,dx\,dy$

4. Evaluate the ordinary improper integral $\int_0^\pi \sin(6x)/\sin x\,dx$.

   _____

5. In $\int_{-\infty}^{\infty} x^2 e^{-x^2} dx$ by parts, the boundary term $-\frac{1}{2}xe^{-x^2}$ vanishes at both infinities.

   ☐ True ☐ False

6. For $I_n(x) = \int_1^x (\ln t)^n\,dt$, give $I_0(3)$.

   _____

7. Evaluate $\int_1^\infty (1+\ln x)/x^2\,dx$.

   _____

8. If two iterated improper integrals of a sign-changing function are both finite, they must be equal.

   ☐ True ☐ False

9. For continuous f on $0 \le x \le 1$, $x^2 \le y \le x$, what is the reversed integral?

   ☐ $\int_0^1 \int_y^{\sqrt{y}} f(x,y)\,dx\,dy$

   ☐ $\int_0^1 \int_{\sqrt{y}}^y f(x,y)\,dx\,dy$

   ☐ $\int_0^1 \int_{y^2}^y f(x,y)\,dx\,dy$

   ☐ $\int_0^1 \int_0^y f(x,y)\,dx\,dy$

10. If f is integrable on $[a, b]$ and $f(a+b-x) = -f(x)$ throughout the interval, then $\int_a^b f(x)\,dx = 0$.

☐ True ☐ False

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Answers ·

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1. **1**

   Parts gives $[xe^x]_0^1 - \int_0^1 e^x\,dx = e - (e-1) = 1$. Equivalently $[e^x(x-1)]_0^1 = 0 - (-1) = 1$.

2. **0.23**

   The continuous integrand is on $0 \le x \le y \le 1$. Reverse to $\int_0^1 \int_0^y \sin(y^2)\,dx\,dy = \int_0^1 y\sin(y^2)\,dy = (1-\cos 1)/2 \approx 0.229849$, hence 0.230.

3. $\int_0^1 \int_{2y}^2 f\,dx\,dy$

   The outer y range is $0 \le y \le 1$. At fixed y, solve $y \le x/2$ to obtain $2y \le x \le 2$.

4. **0**

   The endpoint limits are 6 and -6, so the singularities are removable. Reflection $x \mapsto \pi - x$ changes the sign and preserves the interval; the integral is zero.

5. **True**

   True. On each finite interval the boundary product is $-xe^{-x^2}/2$. Its limit is zero at both infinities. First establish the two half-line integrals; evenness then gives the full-line value $\sqrt{\pi}/2$.

6. **2**

   The base case is $I_0(x) = x - 1$, so $I_0(3) = 2$. The reduction for $n \ge 1$ must not be used with $n = 0$.

7. **2**

   On $[1, R]$, parts gives boundary difference $1 - (1 + \ln R)/R$ and remaining integral $1 - 1/R$. Their sum tends to 2.

8. **False**

   False. On the positive unit square the lesson\'s function $(x^2 - y^2)/(x^2 + y^2)^2$ gives $\pi/4$ and $-\pi/4$. Absolute integrability fails near the origin.

9. $\int_0^1 \int_y^{\sqrt{y}} f(x, y)\,dx\,dy$

   From $y \le x$ obtain $x \ge y$; from $x^2 \le y$ and $x \ge 0$ obtain $x \le \sqrt{y}$. The outer range is $0 \le y \le 1$.

10. **True**

    True. Reflection maps the interval onto itself and makes the integral equal its negative. Integrability is part of the hypothesis.