

Starter Quiz

GRE Subject Test · GRE Mathematics
 Name: _____ Score: _____

- $x = u + 2v$, $y = 3u + 4v$. Find $\partial u / \partial x$ (with y fixed).

- For $x = u + 2v$, $y = 3u + 4v$, find $\partial v / \partial y$ with x fixed.

- At $(u, v) = (1, 2)$, the Jacobian determinant of $(x, y) = (uv, u + v)$ is

☐ 0
 ☐ 1
 ☐ -1
 ☐ 3
- For a map of two variables, $\partial u / \partial x$ always equals $1 / (\partial x / \partial u)$.

☐ True ☐ False
- A zero Jacobian determinant does not by itself rule out a continuous local inverse.

☐ True ☐ False
- For $J = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$, what is the Jacobian determinant?

☐ -3
 ☐ -1/3
 ☐ $\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$
☐ 3
- For $F(u, v) = (u^2 + v, u - v)$, the inverse derivative obtained from $J(1, 0)$ is evaluated at which output point?

☐ (0,1)
 ☐ (1,1)
 ☐ (1,0)
 ☐ (-2,-3)
- The map $x = u^2 + v$, $y = u - v$ sends $(1, 0)$ to $(1, 1)$. What is the other source value of u for that same output?

- The map $F(u, v) = (u^3, v)$ has a differentiable two-sided inverse at output $(0, 0)$.

☐ True ☐ False
- For $x = 2u + v$, $y = 3v$, the inverse partial u_x with y fixed equals $1 / x_u$ with v fixed.

☐ True ☐ False

1. **-2**

$\det J = -2; u_x = g_v / \det J = 4 / (-2).$

2. **-0.5**

By the inverse function theorem, $J^{-1} = (1/\det J) \cdot \text{adj } J$, so $\partial v/\partial y = f_u / \det J$. With $f_u = 1$ and $\det J = -2$ this is -0.5 .

3. **1**

$v \cdot 1 - u \cdot 1 = 2 - 1.$

4. **False**

The forward derivative holds v fixed, while the inverse derivative holds y fixed. In $x=u+v, y=u+2v, x_u=1$ but $u_x=2$. Special triangular or decoupled maps can satisfy a reciprocal relation.

5. **True**

The map $F(u,v)=(u^3,v)$ has continuous inverse (cube root of x,y), despite determinant zero at $u=0$. That inverse is not differentiable there. A differentiable two-sided inverse would force an invertible derivative by the chain rule.

6. **−3**

The determinant is the scalar $2(-1)-1(1)=-3$; J itself is a matrix.

7. **(1,1)**

$F(1,0)=(1,1)$. The derivative of the inverse belongs to the corresponding output point, on the branch through source $(1,0)$.

8. **-2**

Elimination gives $u^2+u=2$, so $(u-1)(u+2)=0$. The other u is -2 and $v=u-y=-3$.

9. **False**

Its continuous inverse has first component cube root of x , which has no finite x derivative at zero. Also $DG \cdot DF=I$ is impossible when DF is singular.

10. **True**

Holding y fixes v , so $2u+v=x$ gives $u_x=1/2$. The forward derivative x_u is 2 ; this triangular map is a valid special case.