27. C.10 · Implicit differentiation and the inverse Jacobian

Starter Quiz ·

GRE Subject Test · GRE Mathematics Name: _____ Score: _____

1. $x = u + 2v$, $y = 3u + 4v$. Find $\partial u/\partial x$ (with y fixed).

   _____

2. For $x = u + 2v$, $y = 3u + 4v$, find $\partial v/\partial y$ with x fixed.

   _____

3. At $(u, v) = (1, 2)$, the Jacobian determinant of $(x, y) = (uv, u + v)$ is

   ☐ 0

   ☐ 1

   ☐ -1

   ☐ 3

4. For a map of two variables, $\partial u/\partial x$ always equals $1/(\partial x/\partial u)$.

   ☐ True ☐ False

5. A zero Jacobian determinant does not by itself rule out a continuous local inverse.

   ☐ True ☐ False

6. For $J = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$, what is the Jacobian determinant?

   ☐ -3

   ☐ $-1/3$

   ☐ $\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$

   ☐ 3

7. For $F(u, v) = (u^2 + v, u - v)$, the inverse derivative obtained from $J(1, 0)$ is evaluated at which output point?

   ☐ (0,1)

   ☐ (1,1)

   ☐ (1,0)

   ☐ $(-2,-3)$

8. The map $x = u^2 + v$, $y = u - v$ sends $(1, 0)$ to $(1, 1)$. What is the other source value of u for that same output?

9. The map $F(u, v) = (u^3, v)$ has a differentiable two-sided inverse at output $(0, 0)$.

☐ True ☐ False

10. For $x = 2u + v$, $y = 3v$, the inverse partial u_x with y fixed equals $1/x_u$ with v fixed.

☐ True ☐ False

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Answers ·

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1. **-2**

   $\det J = -2$; $u_x = g_v / \det J = 4/(-2)$.

2. **-0.5**

   By the inverse function theorem, $J^{-1} = (1/\det J) \cdot \text{adj } J$, so $\partial v/\partial y$ = f_u / det J. With f_u = 1 and det J = −2 this is −0.5.

3. 1

   $v \cdot 1 - u \cdot 1 = 2 - 1$.

4. **False**

   The forward derivative holds v fixed, while the inverse derivative holds y fixed. In x=u+v,y=u+2v, x_u=1 but u_x=2. Special triangular or decoupled maps can satisfy a reciprocal relation.

5. **True**

   The map F(u,v)=(u^3,v) has continuous inverse (cube root of x,y), despite determinant zero at u=0. That inverse is not differentiable there. A differentiable two-sided inverse would force an invertible derivative by the chain rule.

6. **−3**

   The determinant is the scalar 2(-1)-1(1)=-3; J itself is a matrix.

7. **(1,1)**

   F(1,0)=(1,1). The derivative of the inverse belongs to the corresponding output point, on the branch through source (1,0).

8. **-2**

   Elimination gives u^2+u=2, so (u-1)(u+2)=0. The other u is -2 and v=u-y=-3.

9. **False**

   Its continuous inverse has first component cube root of x, which has no finite x derivative at zero. Also DG DF=I is impossible when DF is singular.

10. **True**

    Holding y fixes v, so 2u+v=x gives u_x=1/2. The forward derivative x_u is 2; this triangular map is a valid special case.