

Starter Quiz ·

GRE Subject Test · GRE Mathematics Name: _____ Score: _____

1. How many non-negative integer solutions does $x_1 + x_2 + x_3 = 8$ have?

2. How many positive integer solutions does $x_1 + x_2 + x_3 + x_4 = 9$ have?

3. Five different books are given to 3 students (a student may get none). How many ways?

☐ $3^5 = 243$
☐ $\binom{7}{2} = 21$
☐ $\binom{5}{3} = 10$
☐ $5^3 = 125$
4. Use the stated pre-test odd-sum loop with $S = 0$, $m = 0$, $B = 1$ and threshold $N = 30$. What is S at exit?

5. A loop invariant shows what is true when a loop ends, but termination needs a separate argument.

☐ True ☐ False
6. How many allocations put zero identical tokens into four named boxes, with empty boxes allowed?

7. Count integer solutions of $x_1 + x_2 + x_3 = 8$ with $x_1 \geq 2$, $x_2 \geq 1$, $x_3 \geq 0$.

8. Moving $B \leftarrow B+2$ before $S \leftarrow S+B$ preserves the original loop-head invariant $S = m^2$ with the same initialization.

☐ True ☐ False
9. Reset $S=0$, $m=0$ and $B=1$ before each run of the stated odd-sum loop. Which outputs correspond to thresholds 2 and 5?

☐ 2 and 5
☐ 1 and 4
☐ 4 and 5

10. Eleven identical tokens go into four named boxes. How many allocations have exactly two empty boxes?

1. **45**

Stars and bars: 8 stars and 2 bars, so $C(8 + 2, 2) = C(10, 2) = 45$ non-negative solutions.

2. **56**

Set $y_i = x_i - 1 \geq 0$, so $y_1 + \dots + y_4 = 5$. Stars and bars: $C(5 + 3, 3) = C(8, 3) = 56$.

3. **$3^5 = 243$**

Each distinct book picks a student.

4. **36**

The loop-head sums are 0,1,4,9,16,25,36. Since $25 < 30 \leq 36$, it exits after six additions with $S=36$, $m=6$ and next addend $B=13$.

5. **True**

An established and preserved invariant, combined with a false guard, proves the exit claim if the loop stops. A separate argument, such as the decreasing nonnegative integer $\text{ceil}(\text{sqrt}(N)) - m$ here, proves that it does stop.

6. **1**

There is exactly one tuple: (0,0,0,0). The stars-and-bars count is $C(3,3)=1$.

7. **21**

Subtract lower bounds 2,1,0, leaving three nonnegative counts totaling 5. The count is $C(7,2)=21$.

8. **False**

The first complete pass then gives $S=3$, $m=1$, $B=3$. The original equation $S=m^2$ already fails; the new sum is $m(m+2)$.

9. **4 and 9**

The first squares at least 2 and 5 are 4 and 9. The output 5 on the second run would result from retaining $B=5$ after resetting only S .

10. **60**

Choose the two empty boxes in $C(4,2)=6$ ways. Split 11 positively between the remaining two in $C(10,1)=10$ ways. Multiply to obtain 60.