26. T.11 · Weak compositions, loop invariants and flowchart tracing

Starter Quiz ·

GRE Subject Test · GRE Mathematics Name: _____ Score: _____

1. How many non-negative integer solutions does $x_1 + x_2 + x_3 = 8$ have?

   _____

2. How many positive integer solutions does $x_1 + x_2 + x_3 + x_4 = 9$ have?

   _____

3. Five different books are given to 3 students (a student may get none). How many ways?

   ☐ $3^5 = 243$

   ☐ $\binom{7}{2} = 21$

   ☐ $\binom{5}{3} = 10$

   ☐ $5^3 = 125$

4. Use the stated pre-test odd-sum loop with $S = 0$, $m = 0$, $B = 1$ and threshold $N = 30$. What is S at exit?

   _____

5. A loop invariant shows what is true when a loop ends, but termination needs a separate argument.

   ☐ True ☐ False

6. How many allocations put zero identical tokens into four named boxes, with empty boxes allowed?

   _____

7. Count integer solutions of $x_1 + x_2 + x_3 = 8$ with $x_1 \geq 2$, $x_2 \geq 1$, $x_3 \geq 0$.

   _____

8. Moving B ← B+2 before S ← S+B preserves the original loop-head invariant $S = m^2$ with the same initialization.

   ☐ True ☐ False

9. Reset S=0, m=0 and B=1 before each run of the stated odd-sum loop. Which outputs correspond to thresholds 2 and 5?

   ☐ 2 and 5

   ☐ 1 and 4

   ☐ 4 and 5

☐ 4 and 9

10. Eleven identical tokens go into four named boxes. How many allocations have exactly two empty boxes?

_____

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Answers ·

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1. **45**

   Stars and bars: 8 stars and 2 bars, so C(8 + 2, 2) = C(10, 2) = 45 non-negative solutions.

2. **56**

   Set $y_i = x_i - 1 \geq 0$, so $y_1 + \ldots + y_4 = 5$. Stars and bars: C(5 + 3, 3) = C(8, 3) = 56.

3. $3^5 = 243$

   Each distinct book picks a student.

4. **36**

   The loop-head sums are 0,1,4,9,16,25,36. Since 25<30≤36, it exits after six additions with S=36, m=6 and next addend B=13.

5. **True**

   An established and preserved invariant, combined with a false guard, proves the exit claim if the loop stops. A separate argument, such as the decreasing nonnegative integer ceil(sqrt(N))-m here, proves that it does stop.

6. **1**

   There is exactly one tuple: (0,0,0,0). The stars-and-bars count is C(3,3)=1.

7. **21**

   Subtract lower bounds 2,1,0, leaving three nonnegative counts totaling 5. The count is C(7,2)=21.

8. **False**

   The first complete pass then gives S=3, m=1, B=3. The original equation $S=m^2$ already fails; the new sum is m(m+2).

9. **4 and 9**

   The first squares at least 2 and 5 are 4 and 9. The output 5 on the second run would result from retaining B=5 after resetting only S.

10. **60**

    Choose the two empty boxes in C(4,2)=6 ways. Split 11 positively between the remaining two in C(10,1)=10 ways. Multiply to obtain 60.