

25. C.9 · Integrating factors and nonhomogeneous differential equations

Starter Quiz ·

GRE Subject Test · GRE Mathematics

Name: _____ Score: _____

1. An integrating factor for $y' + \frac{2}{x}y = x$ on $x > 0$ is

☐ e^{2x}

☐ x^2

☐ $2 \ln x$

☐ $1/x^2$

2. $y'' + y' - 6y = 12$ has a constant particular solution y_p . Find it.

3. The correct trial for $y'' - y = e^x$ is

☐ Ae^x

☐ Ae^{-x}

☐ Axe^x

☐ Ax^2e^x

4. $y' + y = 1$, $y(0) = 3$. Find $y(\ln 2)$.

5. The constants C_1, C_2 should be fitted to the initial conditions before adding y_p .

☐ True ☐ False

6. For $2y' + 4xy = 2x$, what is $p(x)$ in the normalized form $y' + p(x)y = q(x)$?

☐ 2

☐ x

☐ $4x$

☐ $2x$

7. If e^{x^2} is an integrating factor for $y' + 2xy = x$, then $-3e^{x^2}$ is also an integrating factor.

☐ True ☐ False

8. The formula $y = x^2/4$ solves $y' + (2/x)y = x$ on the whole real line because it has a value at zero.

☐ True ☐ False

9. For $y'' - 3y' + 2y = x$, let $y_p = x/2 + B$. Find B .

10. Choose the trial for $y'' - 2y' + y = e^x$ that includes the required resonance correction.

☐ Axe^x

☐ Ae^x

☐ Ax^2e^x

☐ $A \sin x + B \cos x$

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Answers ·

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1. x^2

On $x > 0$, an antiderivative of $2/x$ is $2 \ln x$, so choose $\mu = x^2$. Then $(x^2 y)' = x^3$. Nonzero constant multiples of x^2 also work on this interval.

2. **-2**

Try $y_p = A$: then $y_p' = y_p'' = 0$, so $-6A = 12$ and $A = -2$.

3. Axe^x

The root 1 is simple, so multiply Ae^x by x . Substitution gives $L[Axe^x] = 2Ae^x$ and $A = 1/2$. The trial is Axe^x ; its fitted coefficient is $1/2$.

4. **2**

$y = 1 + 2e^{-x}$; at $\ln 2$ this is $1 + 1$.

5. **False**

The initial values belong to $y = y_h + y_p$ and its derivative. Include y_p and y_p' in the equations for C_1, C_2 ; fitting y_h alone generally gives different initial values.

6. $2x$

Divide every term by 2: $y' + 2xy = x$. The coefficient p is $2x$.

7. **True**

A nonzero constant multiple has the same logarithmic derivative $\mu'/\mu = 2x$ and produces the same product identity.

8. **False**

The equation itself is undefined at zero. The formula solves it on either side, not on an interval containing zero.

9. **0.75**

The constant term on the left is $2B - 3/2$. Set it to zero, giving $B = 3/4$.

10. Ax^2e^x

The characteristic polynomial is $(m-1)^2$. Multiply the exponential trial by x^2 , because both e^x and xe^x are homogeneous.