

24. T.10 · Vector geometry, projections and oriented area

Starter Quiz ·

GRE Subject Test · GRE Mathematics

Name: _____ Score: _____

1. Find the area of the triangle with vertices $(0, 0, 0)$, $(1, 2, 0)$ and $(3, 1, 0)$.

2. Find the distance from $(1, 1, 1)$ to the plane $x + 2y + 2z = 14$.

3. The vector projection of $(2, 3)$ onto $(1, 1)$ is
☐ $(5, 5)$
☐ $(1, 1)$
☐ $(2, 3)$
☐ $(2.5, 2.5)$
4. $(1, 2, -1)$ and $(3, -1, 1)$ are perpendicular.
☐ True ☐ False
5. For $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$, if $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = 0$, the three vectors are linearly _____.

6. The zero vector makes a right angle with every nonzero vector because their dot product is zero.
☐ True ☐ False
7. Find the signed scalar component of $(-2, 1)$ in the direction $(1, 0)$.

8. Find the nearest point to $(4, 3)$ on the line $(1, 1) + t(1, 1)$.
☐ $(4, 3)$
☐ $(5/2, 5/2)$
☐ $(1, 1)$
☐ $(7/2, 7/2)$
9. If $\mathbf{u} \times \mathbf{v} = (2, -1, 4)$, what is $\mathbf{v} \times \mathbf{u}$?
☐ $\sqrt{21}$
☐ $(-2, -1, -4)$
☐ $(2, -1, 4)$

☐ $(-2, 1, -4)$

10. The equation $0x + 0y + 0z = 0$ describes one plane with normal $(0, 0, 0)$.

☐ True ☐ False

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Answers ·

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1. **2.5**

Use edges $(1,2,0)$ and $(3,1,0)$ from the origin. Their cross product is $(0,0,-5)$; its length is 5, so the triangle area is $5/2$.

2. **3**

The nonzero normal is $(1,2,2)$, of length 3. The signed numerator is $1+2+2-14=-9$; take its absolute value and divide by 3. The nearest point is $(2,3,3)$.

3. **$(2.5, 2.5)$**

The coefficient is $(2+3)/(1+1)=5/2$. Multiply the target vector $(1,1)$ by it to obtain $(5/2, 5/2)$; the remainder $(-1/2, 1/2)$ is perpendicular to $(1,1)$.

4. **True**

Both vectors are nonzero and their dot product is $3-2-1=0$, so they are perpendicular.

5. **dependent**

The scalar triple product is the determinant of the three vectors. It vanishes exactly when they are linearly dependent; the spanned parallelepiped has zero volume.

6. **False**

The zero vector has no direction and no defined angle. The angle formula requires both lengths to be nonzero.

7. **-2**

Divide the dot product -2 by the target length 1. The signed component is -2; its absolute value 2 is the projection length.

8. **$(7/2, 7/2)$**

Project displacement $(3,2)$ onto $(1,1)$: coefficient $5/2$. Add $(1,1)$ back to obtain $(7/2, 7/2)$. The residual $(1/2, -1/2)$ is perpendicular to the line.

9. **$(-2, 1, -4)$**

Swapping the order reverses every component of the vector; its length stays the same.

10. **False**

Every point in space satisfies this equation. A plane equation needs a nonzero normal.