20. T.7 · Set images, equivalence relations and logical negation

Starter Quiz ·

GRE Subject Test · GRE Mathematics Name: _____ Score: _____

1. For every function f and sets A, B in its domain, which always holds?

   ☐ $f(A \cap B) = f(A) \cap f(B)$

   ☐ $f(A \setminus B) = f(A) \setminus f(B)$

   ☐ $f(A) \cap f(B) \subseteq f(A \cap B)$

   ☐ $f(A \cup B) = f(A) \cup f(B)$

2. How many equivalence classes does "same remainder mod 6" have on $\mathbb{Z}$?

   _____

3. On $\mathbb{Z}$, the relation "$x - y$ is odd" is

   ☐ reflexive but not symmetric

   ☐ transitive and reflexive

   ☐ an equivalence relation

   ☐ symmetric but not reflexive

4. The negation of "for every $\varepsilon > 0$ there is N with $|a_n| < \varepsilon$ for all $n > N$" is

   ☐ For every $\varepsilon > 0$ and every cutoff N, some $n > N$ has $|a_n| \geq \varepsilon$.

   ☐ There is $\varepsilon > 0$ such that for every cutoff N, some $n > N$ has $|a_n| \geq \varepsilon$.

   ☐ There are $\varepsilon > 0$ and a cutoff N such that every $n > N$ has $|a_n| \geq \varepsilon$.

   ☐ For every $\varepsilon > 0$, some cutoff N makes every $n > N$ satisfy $|a_n| \geq \varepsilon$.

5. $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$ for every function f.

   ☐ True ☐ False

6. $P \Rightarrow Q$ is false only when P is true and Q is _____.

   _____

7. Let a, b be distinct and $f(a) = f(b) = L$. For $A = \{a\}$ and $B = \{b\}$, what are $f(A \cap B)$ and $f(A) \cap f(B)$?

   ☐ $\{L\}$ and $\varnothing$

   ☐ Both are $\varnothing$.

   ☐ $\varnothing$ and $\{L\}$

   ☐ Both are $\{L\}$.

8. For $f(x) = x^2$ on $\mathbb{R}$ and $A = \{-2\}$, how many elements belong to $f^{-1}(f(A))$?

_____

9. Let $X = \{a, b, c\}$, $Y = \{L, M, N\}$, $f(a) = f(b) = L$, $f(c) = M$, and $C = \{L, N\}$. Find $f(f^{-1}(C))$.

   ☐ $\{a, b\}$

   ☐ $\{L, M\}$

   ☐ $\{L, N\}$

   ☐ $\{L\}$

10. The relation $xy > 0$ is an equivalence relation on all of $\mathbb{R}$.

    ☐ True ☐ False

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Answers ·

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1. $f(A \cup B) = f(A) \cup f(B)$

   Union equality holds for every map. The other equalities/inclusion become valid for every pair under injectivity, but can fail for a map with a collision. A particular pair can still give equality without injectivity.

2. **6**

   Two integers are related when they leave the same remainder on division by 6. The possible remainders 0, 1, …, 5 give exactly 6 classes.

3. **symmetric but not reflexive**

   It fails reflexivity because x − x = 0 is even. It is symmetric (x − y odd ⇒ y − x odd) but not transitive: 1 − 2 and 2 − 3 are odd, yet 1 − 3 is even.

4. **There is $\varepsilon > 0$ such that for every cutoff N, some $n > N$ has $|a_n| \geq \varepsilon$.**

   Flip ∀ε, ∃N, ∀n>N to ∃ε, ∀N, ∃n>N, in that order. Failure of a strict inequality includes equality. Alternating zeros and ones fails at ε=1/2 but has late zeros, and ε=2 never fails.

5. **True**

   An input maps into C ∩ D exactly when it maps into both C and D; injectivity is unnecessary.

6. **false**

   Truth table of implication.

7. **$\varnothing$ and $\{L\}$**

   The input intersection is empty, but both image sets contain L.

8. **2**

   f(A)={4}, whose preimage is {-2,2}. The round trip can add inputs.

9. $\{L\}$

   Only L is in both C and the image {L,M}. No input maps to N.

10. **False**

    At x=0, x^2 is not positive, so reflexivity fails. Removing zero gives an equivalence relation with two sign classes.