

Starter Quiz

1. Which map is a bijection?

☐ $x \mapsto x^3$ from $\mathbb{R}$ to $\mathbb{R}$

☐ $x \mapsto x^2$ from $\mathbb{R}$ to $\mathbb{R}$

☐ $x \mapsto x^2$ from $[0, \infty)$ to $\mathbb{R}$

☐ $x \mapsto e^x$ from $\mathbb{R}$ to $\mathbb{R}$
2. $f(x) = x^2 + 4x$ on $x \geq -2$. Find $f^{-1}(5)$.

3. $g(x) = \frac{1}{1-x}$. Find $g^{10}(2)$, the tenth iterate.

4. For a bijective function f , the graph of f^{-1} is the reflection of its graph in the line $y = x$.

☐ True ☐ False
5. A self-map $f : A \rightarrow A$ with $f(f(x)) = x$ for all $x \in A$ is called an _____.

6. How is $x \mapsto x^2$ classified as a map $\mathbb{R} \rightarrow \mathbb{R}$?

☐ Surjective only

☐ Injective only

☐ Bijective

☐ Neither injective nor surjective
7. For $f(x) = (x - 2)^2 - 5$ restricted to $x \leq 2$, find $f^{-1}(4)$.

8. If $f : A \rightarrow B$ and $h : B \rightarrow A$ satisfy $f \circ h = \text{id}_B$, then f must be bijective.

☐ True ☐ False
9. Let $a(x) = x + 1$ and $b(x) = x^2$. Evaluate $(a \circ b)(2)$.

10. A strictly increasing self-map of an interval satisfying $f \circ f = \text{id}$ must be the identity.

☐ True ☐ False

Answers

1. $x \mapsto x^3$ **from $\mathbb{R}$ to $\mathbb{R}$**

$e^{\wedge}x$ misses negatives; x^2 fails injectivity or surjectivity.
2. **1**

$x^2 + 4x - 5 = 0$ gives 1 or -5 ; only 1 is allowed.
3. **-1**

Period 3, $10 \equiv 1$: $g(2) = -1$.
4. **True**

True. Each point (a, b) on f becomes (b, a) on f^{-1} . Replacing y by $1/y$ gives the reciprocal, a different function.
5. **involution**

It is its own inverse.
6. **Neither injective nor surjective**

Inputs 1 and -1 collide, so it is not injective. Negative codomain values are missed, so it is not surjective.
7. **-1**

The allowed branch is $2 - \sqrt{y+5}$. At $y=4$ this gives -1; the other root 5 is not in the original domain.
8. **False**

Squaring from $\mathbb{R}$ to $[0, \infty)$ and $h(y)=\sqrt{y}$ satisfy this identity. But squaring is not injective, and the reverse composition is $|x|$, not x on all $\mathbb{R}$.
9. **5**

Apply b first: $b(2)=4$, then $a(4)=5$. The opposite composition gives 9.
10. **True**

If $f(x)>x$, monotonicity gives $f(f(x))>f(x)$, contradicting $f(f(x))=x$. The case $f(x)<x$ gives the reverse contradiction.