

## 19. T.6 · Functions, inverse branches and composition

### Starter Quiz ·

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GRE Subject Test · GRE Mathematics

Name: \_\_\_\_\_ Score: \_\_\_\_\_

1. Which map is a bijection?

☐  $x \mapsto x^3$  from  $\mathbb{R}$  to  $\mathbb{R}$

☐  $x \mapsto x^2$  from  $\mathbb{R}$  to  $\mathbb{R}$

☐  $x \mapsto x^2$  from  $[0, \infty)$  to  $\mathbb{R}$

☐  $x \mapsto e^x$  from  $\mathbb{R}$  to  $\mathbb{R}$

2.  $f(x) = x^2 + 4x$  on  $x \geq -2$ . Find  $f^{-1}(5)$ .

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3.  $g(x) = \frac{1}{1-x}$ . Find  $g^{10}(2)$ , the tenth iterate.

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4. For a bijective function  $f$ , the graph of  $f^{-1}$  is the reflection of its graph in the line  $y = x$ .

☐ True      ☐ False

5. A self-map  $f : A \rightarrow A$  with  $f(f(x)) = x$  for all  $x \in A$  is called an \_\_\_\_\_.

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6. How is  $x \mapsto x^2$  classified as a map  $\mathbb{R} \rightarrow \mathbb{R}$ ?

☐ Surjective only

☐ Injective only

☐ Bijective

☐ Neither injective nor surjective

7. For  $f(x) = (x - 2)^2 - 5$  restricted to  $x \leq 2$ , find  $f^{-1}(4)$ .

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8. If  $f : A \rightarrow B$  and  $h : B \rightarrow A$  satisfy  $f \circ h = \text{id}_B$ , then  $f$  must be bijective.

☐ True      ☐ False

9. Let  $a(x) = x + 1$  and  $b(x) = x^2$ . Evaluate  $(a \circ b)(2)$ .

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10. A strictly increasing self-map of an interval satisfying  $f \circ f = \text{id}$  must be the identity.

☐ True      ☐ False

## 19. T.6 · Functions, inverse branches and composition

### Answers ·

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1.  $x \mapsto x^3$  **from  $\mathbb{R}$  to  $\mathbb{R}$**

$e^x$  misses negatives;  $x^2$  fails injectivity or surjectivity.

2. **1**

$x^2 + 4x - 5 = 0$  gives 1 or  $-5$ ; only 1 is allowed.

3. **-1**

Period 3,  $10 \equiv 1$ :  $g(2) = -1$ .

4. **True**

True. Each point  $(a, b)$  on  $f$  becomes  $(b, a)$  on  $f^{-1}$ . Replacing  $y$  by  $1/y$  gives the reciprocal, a different function.

5. **involution**

It is its own inverse.

6. **Neither injective nor surjective**

Inputs 1 and  $-1$  collide, so it is not injective. Negative codomain values are missed, so it is not surjective.

7. **-1**

The allowed branch is  $2 - \sqrt{y+5}$ . At  $y=4$  this gives  $-1$ ; the other root 5 is not in the original domain.

8. **False**

Squaring from  $\mathbb{R}$  to  $[0, \infty)$  and  $h(y) = \sqrt{y}$  satisfy this identity. But squaring is not injective, and the reverse composition is  $|x|$ , not  $x$  on all  $\mathbb{R}$ .

9. **5**

Apply  $b$  first:  $b(2)=4$ , then  $a(4)=5$ . The opposite composition gives 9.

10. **True**

If  $f(x) > x$ , monotonicity gives  $f(f(x)) > f(x)$ , contradicting  $f(f(x)) = x$ . The case  $f(x) < x$  gives the reverse contradiction.