18. C.7 · Coordinate changes and vector integral theorems

Starter Quiz ·

GRE Subject Test · GRE Mathematics Name: _____ Score: _____

1. The outward flux of $\mathbf{F} = (x, y, z)$ through the unit sphere is $k\pi$. Find k.

   _____

2. The counter-clockwise circulation of $(-y, x)$ around the circle of radius 3 is $k\pi$. Find k.

   _____

3. The spherical volume element (φ measured from the z-axis) is

   ☐ $\rho\, d\rho\, d\varphi\, d\theta$

   ☐ $\rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

   ☐ $\sin\varphi\, d\rho\, d\varphi\, d\theta$

   ☐ $\rho^2 \cos\varphi\, d\rho\, d\varphi\, d\theta$

4. A curl-free field on $\mathbb{R}^2 \setminus \{0\}$ is conservative.

   ☐ True ☐ False

5. $\mathbf{F} = \nabla(x^2 y)$. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ along any path from $(0,0)$ to $(2,3)$.

   _____

6. The integral of $x^2 + y^2$ over $1 \le x^2 + y^2 \le 4$ is $k\pi$. Find k.

   _____

7. The volume of $0 \le z \le 4 - x^2 - y^2$ is $k\pi$. Find k.

   _____

8. If a field is conservative on an open connected domain, that domain must be simply connected.

   ☐ True ☐ False

9. The integral of $x^2 + y^2 + z^2$ over the ball of radius two is $k\pi$. Find k.

   _____

10. For $\mathbf{F} = (-y/2, x/2, z)$, the circulation around $x^2 + y^2 = 4$, $z = 0$, clockwise from above, is $k\pi$. Find k.

    _____

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Answers ·

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1. **4**

   By divergence, outward flux is divergence 3 times unit-ball volume $4\pi/3$. Thus flux is 4π and $k = 4$.

2. **18**

   Green gives scalar curl $Q_x - P_y = 1 - (-1) = 2$. The radius-three disk has area 9π, so circulation is 18π and $k = 18$.

3. $\rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

   The Jacobian is $\rho^2 \sin\varphi$.

4. **False**

   The angular field circulates 2π.

5. **12**

   $\varphi(2,3) - \varphi(0,0) = 12$.

6. **7.5**

   The integrand is r^2 and the area factor is r. Thus $2\pi \int_1^2 r^3\, dr = 15\pi/2$, so k = 7.5.

7. **8**

   The projection disk has $0 \le r \le 2$. Integrating the height with the cylindrical factor gives $2\pi \int_0^2 (4 - r^2) r\, dr = 8\pi$.

8. **False**

   On the punctured plane, (2x,2y) has the global potential $x^2 + y^2$. Simple connectedness is sufficient for the general zero-curl implication, not necessary for an individual field to be conservative.

9. **25.6**

   The integrand becomes ρ^2 and dV contributes another $\rho^2 \sin\varphi$. Angular integration gives 4π, and $\int_0^2 \rho^4\, d\rho = 32/5$, so k = 128/5 = 25.6.

10. **-4**

    The upward/counter-clockwise orientation gives 4π. Reversing traversal gives -4π, matching a downward surface normal.