

18. C.7 · Coordinate changes and vector integral theorems

Starter Quiz ·

GRE Subject Test · GRE Mathematics

Name: _____ Score: _____

1. The outward flux of $\mathbf{F} = (x, y, z)$ through the unit sphere is $k\pi$. Find k .

2. The counter-clockwise circulation of $(-y, x)$ around the circle of radius 3 is $k\pi$. Find k .

3. The spherical volume element (φ measured from the z -axis) is
☐ $\rho d\rho d\varphi d\theta$
☐ $\rho^2 \sin \varphi d\rho d\varphi d\theta$
☐ $\sin \varphi d\rho d\varphi d\theta$
☐ $\rho^2 \cos \varphi d\rho d\varphi d\theta$
4. A curl-free field on $\mathbb{R}^2 \setminus \{0\}$ is conservative.
☐ True ☐ False
5. $\mathbf{F} = \nabla(x^2y)$. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ along any path from $(0, 0)$ to $(2, 3)$.

6. The integral of $x^2 + y^2$ over $1 \leq x^2 + y^2 \leq 4$ is $k\pi$. Find k .

7. The volume of $0 \leq z \leq 4 - x^2 - y^2$ is $k\pi$. Find k .

8. If a field is conservative on an open connected domain, that domain must be simply connected.
☐ True ☐ False
9. The integral of $x^2 + y^2 + z^2$ over the ball of radius two is $k\pi$. Find k .

10. For $\mathbf{F} = (-y/2, x/2, z)$, the circulation around $x^2 + y^2 = 4$, $z = 0$, clockwise from above, is $k\pi$. Find k .

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Answers ·

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1. **4**

By divergence, outward flux is divergence 3 times unit-ball volume $4\pi/3$. Thus flux is 4π and $k = 4$.

2. **18**

Green gives scalar curl $Q_x - P_y = 1 - (-1) = 2$. The radius-three disk has area 9π , so circulation is 18π and $k = 18$.

3. $\rho^2 \sin \varphi \, d\rho \, d\varphi \, d\theta$

The Jacobian is $\rho^2 \sin \varphi$.

4. **False**

The angular field circulates 2π .

5. **12**

$\varphi(2, 3) - \varphi(0, 0) = 12$.

6. **7.5**

The integrand is r^2 and the area factor is r . Thus $2\pi \int_1^2 r^3 \, dr = 15\pi/2$, so $k = 7.5$.

7. **8**

The projection disk has $0 \leq r \leq 2$. Integrating the height with the cylindrical factor gives $2\pi \int_0^2 (4-r^2)r \, dr = 8\pi$.

8. **False**

On the punctured plane, $(2x, 2y)$ has the global potential $x^2 + y^2$. Simple connectedness is sufficient for the general zero-curl implication, not necessary for an individual field to be conservative.

9. **25.6**

The integrand becomes ρ^2 and dV contributes another $\rho^2 \sin \varphi$. Angular integration gives 4π , and $\int_0^2 \rho^4 \, d\rho = 32/5$, so $k = 128/5 = 25.6$.

10. **-4**

The upward/counter-clockwise orientation gives 4π . Reversing traversal gives -4π , matching a downward surface normal.