

Starter Quiz
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GRE Subject Test
 ·
 GRE Mathematics
 Name: _____ Score: _____

- For $f = x^2 + xy + y^2$, compute $D = f_{xx}f_{yy} - f_{xy}^2$ at the origin.

- The origin is a critical point of $f = 3x^2 - 6xy + y^2$. It is

☐ a local maximum
 ☐ a saddle
 ☐ undetermined by the test
 ☐ a local minimum
- Maximise xy subject to $x + y = 10$.

- If $D = 0$ at a critical point, the second-derivative test is inconclusive.

☐ True
 ☐ False
- What is the maximum of $x^2 - y^2$ on the square $[-1, 1] \times [-1, 1]$?

- For $f = x + x^2 + y^2$, positive definiteness of the Hessian at the origin proves a local minimum there.

☐ True
 ☐ False
- If a smooth function has a strict minimum at the origin on every straight line through it, then it has a local minimum in the plane.

☐ True
 ☐ False
- If the Lagrange equations have no solution, a constrained extremum cannot exist.

☐ True
 ☐ False
- A function on an unbounded constraint set cannot attain a global maximum.

☐ True
 ☐ False
- What is the global maximum of $x + 2y$ on $x^2 + y^2 = 1$? Give a decimal to three places.

Answers
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- 3**

 $2 \cdot 2 - 1$; with $f_{xx} > 0$, a minimum.
- a saddle**

 $D = 6 \cdot 2 - 36 < 0$.
- 25**

Substitute $y = 10 - x$: $xy = 10x - x^2$, maximised at $x = 5$ (Lagrange gives $y = \lambda = x$). Then $xy = 5 \cdot 5 = 25$.
- True**

True. $x^4 + y^4$ (minimum), $-x^4 - y^4$ (maximum) and $x^4 - y^4$ (saddle) all have $D = 0$ at the origin.
- 1**

At $(\pm 1, 0)$, on the boundary.
- False**

The gradient at the origin is $(1,0)$, so the point is not stationary. On the x -axis, $x+x^2$ takes both signs arbitrarily near zero. The stationary-point hypothesis is essential.
- False**

For $F=(y-x^2)(y-2x^2)$, every fixed line has a strict minimum at zero, but on $y=3x^2/2$ the value is $-x^4/4$. No single neighbourhood is controlled by all separate line checks.
- False**

For $f=x$ under $x^2+y^2=0$, the origin is the only feasible point and is both a global minimum and maximum. The constraint gradient vanishes, so the regular multiplier theorem does not apply.
- False**

The function $-(x^2+y^2)$ on $x+y=6$ has a global maximum -18 at $(3,3)$, despite the unbounded line. One must inspect the objective, not only the domain.
- 2.236**

The maximum is $\sqrt{5} \approx 2.236$ at $(1/\sqrt{5}, 2/\sqrt{5})$. Cauchy–Schwarz gives $|x+2y| \leq \sqrt{5}$ on the unit circle.