

17. C.6 · Multivariable extrema and constrained optimisation

Starter Quiz ·

GRE Subject Test · GRE Mathematics

Name: _____ Score: _____

1. For $f = x^2 + xy + y^2$, compute $D = f_{xx}f_{yy} - f_{xy}^2$ at the origin.

2. The origin is a critical point of $f = 3x^2 - 6xy + y^2$. It is

☐ a local maximum

☐ a saddle

☐ undetermined by the test

☐ a local minimum

3. Maximise xy subject to $x + y = 10$.

4. If $D = 0$ at a critical point, the second-derivative test is inconclusive.

☐ True ☐ False

5. What is the maximum of $x^2 - y^2$ on the square $[-1, 1] \times [-1, 1]$?

6. For $f = x + x^2 + y^2$, positive definiteness of the Hessian at the origin proves a local minimum there.

☐ True ☐ False

7. If a smooth function has a strict minimum at the origin on every straight line through it, then it has a local minimum in the plane.

☐ True ☐ False

8. If the Lagrange equations have no solution, a constrained extremum cannot exist.

☐ True ☐ False

9. A function on an unbounded constraint set cannot attain a global maximum.

☐ True ☐ False

10. What is the global maximum of $x + 2y$ on $x^2 + y^2 = 1$? Give a decimal to three places.

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Answers ·

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1. **3**

$2 \cdot 2 - 1$; with $f_{xx} > 0$, a minimum.

2. **a saddle**

$$D = 6 \cdot 2 - 36 < 0.$$

3. **25**

Substitute $y = 10 - x$: $xy = 10x - x^2$, maximised at $x = 5$ (Lagrange gives $y = \lambda = x$). Then $xy = 5 \cdot 5 = 25$.

4. **True**

True. $x^4 + y^4$ (minimum), $-x^4 - y^4$ (maximum) and $x^4 - y^4$ (saddle) all have $D = 0$ at the origin.

5. **1**

At $(\pm 1, 0)$, on the boundary.

6. **False**

The gradient at the origin is $(1, 0)$, so the point is not stationary. On the x -axis, $x + x^2$ takes both signs arbitrarily near zero. The stationary-point hypothesis is essential.

7. **False**

For $F = (y - x^2)(y - 2x^2)$, every fixed line has a strict minimum at zero, but on $y = 3x^2/2$ the value is $-x^4/4$. No single neighbourhood is controlled by all separate line checks.

8. **False**

For $f = x$ under $x^2 + y^2 = 0$, the origin is the only feasible point and is both a global minimum and maximum. The constraint gradient vanishes, so the regular multiplier theorem does not apply.

9. **False**

The function $-(x^2 + y^2)$ on $x + y = 6$ has a global maximum -18 at $(3, 3)$, despite the unbounded line. One must inspect the objective, not only the domain.

10. **2.236**

The maximum is $\sqrt{5} \approx 2.236$ at $(1/\sqrt{5}, 2/\sqrt{5})$. Cauchy–Schwarz gives $|x + 2y| \leq \sqrt{5}$ on the unit circle.