Starter Quiz ·

GRE Subject Test · GRE Mathematics Name: _____ Score: _____

1. What is the coefficient of x^3 in the Maclaurin series of e^{2x}? Give it to three decimal places.

   _____

2. What is the radius of convergence of $\sum x^n/3^n$?

   _____

3. The interval of convergence of $\sum_{n\geq 1} x^n/n^2$ is

   ☐ $(-1, 1]$

   ☐ $[-1, 1]$

   ☐ $[-1, 1)$

   ☐ $(-1, 1)$

4. Evaluate $\lim_{x\to 0} \dfrac{\cos x - 1}{x^2}$.

   _____

5. Differentiating a power series term by term keeps its radius of convergence.

   ☐ True ☐ False

6. Which statement certifies that $e^{1/2} > 1.63$?

   ☐ An approximation always preserves inequalities.

   ☐ $T_3(1/2) = 79/48 > 1.63$, with positive remainder.

   ☐ Every Taylor remainder is negative.

   ☐ $T_2(1/2) = 1.625 < 1.63$.

7. An error upper bound larger than the tolerance proves that the actual approximation error exceeds the tolerance.

   ☐ True ☐ False

8. What is the left endpoint of the convergence interval of $\sum_{n\geq 1}(x-2)^n/(n3^n)$?

   _____

9. What is the radius of convergence of $\sum_{n\geq 0} x^{2n}$, viewed as a power series in x?

   _____

10. Where does $\sum_{n\geq 0}(-4)^n x^{2n}$ converge?

    ☐ $(-1/2, 1/2)$

    ☐ $(-1, 1)$

    ☐ $[-1/2, 1/2]$

Answers ·

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1. **1.333**

   e^{2x} = Σ (2x)^n/n!, so the x^3 coefficient is $2^3/3! = 8/6 = 4/3 \approx 1.333$.

2. **3**

   The ratio test gives $|x|/3 < 1$.

3. $[-1, 1]$

   At ± 1 the series is dominated by $\Sigma\, 1/n^2$.

4. **-0.5**

   $\cos x - 1 = -x^2/2 + \cdots$.

5. **True**

   True. The radius stays the same; only convergence at the endpoints can change.

6. $T_3(1/2) = 79/48 > 1.63$**, with positive remainder.**

   The cubic polynomial is $79/48 > 1.63$ and its Lagrange remainder is positive. An upper error bound alone cannot supply a lower bound.

7. **False**

   A loose upper bound cannot prove a lower bound on the actual error. A sharper estimate may still meet the tolerance.

8. **-1**

   The centre is 2 and radius 3, so the left endpoint is 2 - 3 = -1. It is included conditionally.

9. **1**

   It is geometric in x^2, so $|x^2| < 1$ means $|x| < 1$. Zero odd coefficients make the successive-coefficient ratio unsuitable.

10. $(-1/2, 1/2)$

    The geometric ratio is $-4x^2$. Its absolute value must be below 1, giving $|x| < 1/2$. At either endpoint the terms are (-1)^n and do not tend to zero.