

15. C.4 · Taylor expansions and power-series endpoints

Starter Quiz ·

GRE Subject Test · GRE Mathematics

Name: _____ Score: _____

1. What is the coefficient of x^3 in the Maclaurin series of e^{2x} ? Give it to three decimal places.

2. What is the radius of convergence of $\sum x^n/3^n$?

3. The interval of convergence of $\sum_{n \geq 1} x^n/n^2$ is

☐ $(-1, 1]$

☐ $[-1, 1]$

☐ $[-1, 1)$

☐ $(-1, 1)$

4. Evaluate $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$.

5. Differentiating a power series term by term keeps its radius of convergence.

☐ True ☐ False

6. Which statement certifies that $e^{1/2} > 1.63$?

☐ An approximation always preserves inequalities.

☐ $T_3(1/2) = 79/48 > 1.63$, with positive remainder.

☐ Every Taylor remainder is negative.

☐ $T_2(1/2) = 1.625 < 1.63$.

7. An error upper bound larger than the tolerance proves that the actual approximation error exceeds the tolerance.

☐ True ☐ False

8. What is the left endpoint of the convergence interval of $\sum_{n \geq 1} (x - 2)^n/(n3^n)$?

9. What is the radius of convergence of $\sum_{n \geq 0} x^{2n}$, viewed as a power series in x ?

10. Where does $\sum_{n \geq 0} (-4)^n x^{2n}$ converge?

- ☐ All real x .
- ☐ $(-1/2, 1/2)$
- ☐ $(-1, 1)$
- ☐ $[-1/2, 1/2]$

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Answers ·

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1. **1.333**

$e^{2x} = \sum (2x)^n/n!$, so the x^3 coefficient is $2^3/3! = 8/6 = 4/3 \approx 1.333$.

2. **3**

The ratio test gives $|x|/3 < 1$.

3. **$[-1, 1]$**

At ± 1 the series is dominated by $\sum 1/n^2$.

4. **-0.5**

$\cos x - 1 = -x^2/2 + \dots$.

5. **True**

True. The radius stays the same; only convergence at the endpoints can change.

6. **$T_3(1/2) = 79/48 > 1.63$, with positive remainder.**

The cubic polynomial is $79/48 > 1.63$ and its Lagrange remainder is positive. An upper error bound alone cannot supply a lower bound.

7. **False**

A loose upper bound cannot prove a lower bound on the actual error. A sharper estimate may still meet the tolerance.

8. **-1**

The centre is 2 and radius 3, so the left endpoint is $2 - 3 = -1$. It is included conditionally.

9. **1**

It is geometric in x^2 , so $|x^2| < 1$ means $|x| < 1$. Zero odd coefficients make the successive-coefficient ratio unsuitable.

10. **$(-1/2, 1/2)$**

The geometric ratio is $-4x^2$. Its absolute value must be below 1, giving $|x| < 1/2$. At either endpoint the terms are $(-1)^n$ and do not tend to zero.