

Starter Quiz ·

1. The closure of $\mathbb{Q}$ in $\mathbb{R}$ is

- ☐ $\mathbb{R}$
- ☐ $\emptyset$
- ☐ $\mathbb{R} \setminus \mathbb{Q}$
- ☐ $\mathbb{Q}$

2. The boundary of $(0, 1) \cup (1, 2)$ in $\mathbb{R}$ is

- ☐ $\{1\}$
- ☐ $[0, 2]$
- ☐ $\{0, 2\}$
- ☐ $\{0, 1, 2\}$

3. Which subsets of $\mathbb{R}$ are connected?

- ☐ $\{5\}$
- ☐ $[0, \infty)$
- ☐ $\mathbb{Q}$
- ☐ $[0, 1] \cup [2, 3]$
- ☐ $(0, 1)$

Tick every correct option.

4. Every compact subset of $\mathbb{R}$ is connected.

- ☐ True
- ☐ False

5. In $X = [0, 2]$, the set $[0, 1)$ is open because it equals X intersected with an _____ subset of $\mathbb{R}$.

6. For $A = [0, 1/2)$ inside $X = [0, 1]$, what is the status of 0 relative to A in X ?

- ☐ Outside the closure.
- ☐ A boundary point.
- ☐ Not a point of X .
- ☐ An interior point.

- ☐ True
- ☐ False

8. Why does the cover $U_n = (1/n, 1)$, $n \geq 2$, of $(0, 1)$ have no finite subcover?

- ☐ The intervals fail to cover any positive point.
- ☐ The set $(0, 1)$ is unbounded.
- ☐ Any infinite cover automatically has no finite subcover.
- ☐ A finite choice with largest index N misses $1/(2N)$.

9. With distance 1 between distinct points, $\mathbb{N}$ is compact because it is complete and bounded.

- ☐ True
- ☐ False

10. A continuous real-valued function on a nonempty compact domain must

- ☐ be constant.
- ☐ have a connected domain.
- ☐ have an interval as its image.
- ☐ attain both a minimum and a maximum.

1. $\mathbb{R}$
Every real number is a limit of rationals.
2. $\{0, 1, 2\}$
Closure $[0, 2]$ minus interior $(0, 1) \cup (1, 2)$.
3. $\{5\}; [0, \infty); (0, 1)$
The connected subsets of $\mathbb{R}$ are intervals, including single points.
4. **False**
False. $\{0, 1\}$ is closed and bounded, hence compact, but it is not connected: the connected subsets of $\mathbb{R}$ are intervals.
5. **open**
For example $(-1, 1)$.
6. **An interior point.**
A sufficiently small ball in X around 0 is $[0, \varepsilon)$, lying inside A . Thus 0 is an interior point in X . In $\mathbb{R}$, however, every neighbourhood also contains negative points, so 0 is a boundary point there.
7. **True**
It includes its only new accumulation point, 0, so it is closed in $\mathbb{R}$. It is also bounded, so Heine–Borel applies. Deleting 0 would destroy closedness and compactness.
8. **A finite choice with largest index N misses $1/(2N)$.**
The sets are nested as n increases. A finite choice has a largest index N and union $(1/N, 1)$, missing the point $1/(2N)$ of $(0, 1)$. A cover can be infinite and still have a finite subcover, so the number of sets alone is not a proof.
9. **False**
It is complete and bounded, but not totally bounded. Balls of radius $1/2$ are singletons, and no finite set of them covers $\mathbb{N}$. Completeness plus ordinary boundedness is insufficient.
10. **attain both a minimum and a maximum.**
Its image is a nonempty compact subset of $\mathbb{R}$, so its minimum and maximum belong to that image. The image need not be an interval unless connectedness is also assumed.