

Starter Quiz

1. On $[0, \frac{1}{2}]$, what is $\sup |x^3 - 0|$?
- _____
2. Which sequence converges uniformly on $[0, 1]$?
- ☐ $x^{1/n}$
- ☐ $nx(1 - x)^n$
- ☐ x^n
- ☐ x^n/n
3. For $\sum n/2^n$, the ratio test gives
- ☐ limit 0, inconclusive
- ☐ limit 1, inconclusive
- ☐ limit 2, so it diverges
- ☐ limit 1/2, so the series converges
4. A uniform limit of continuous functions is continuous.
- ☐ True ☐ False
5. $\sum (-1)^n/n$ converges, but not absolutely: it converges _____.
- _____
6. Every Cauchy sequence of rational numbers converges to a rational number.
- ☐ True ☐ False
7. If $a_n \rightarrow 0$, then $\sum_{n=1}^\infty a_n$ must converge.
- ☐ True ☐ False
8. At which $x \geq 0$ does $5xe^{-5x}$ attain its maximum?
- _____
9. Which series is uniformly convergent on $[-1, 1]$ by the M-test with a summable bound?
- ☐ $\sum_{n=1}^\infty nx^n$
- ☐ $\sum_{n=1}^\infty x^n/n$
- ☐ $\sum_{n=1}^\infty x^n/n^2$

10. A triangular h_n has base $2/n$ and height n on $[0, 1]$, for $n \geq 2$. What is $\int_0^1 h_n(x) \, dx$?

1. **0.125**

x^3 increases on $[0, \frac{1}{2}]$, so the supremum of $|x^3|$ is at $x = \frac{1}{2}$: $(\frac{1}{2})^3 = 1/8 = 0.125$.

2. **x^n/n**

The sequence x^n/n has supremum error $1/n$, so converges uniformly to zero. For x^n the error supremum is 1. For $nx(1-x)^n$ the maximum is $(n/(n+1))^{n+1}$, tending to $1/e$ rather than zero. For $x^{1/n}$, the pointwise limit is 0 at $x = 0$ and 1 for $x > 0$, so the discontinuous limit rules out uniform convergence of these continuous functions.

3. **limit 1/2, so the series converges**

$$\frac{n+1}{2n} \rightarrow \frac{1}{2}.$$

4. **True**

True. If continuous $f_n \rightarrow f$ uniformly, an $\varepsilon/3$ argument shows f is continuous. Pointwise limits need not be: x^n on $[0, 1]$ has a discontinuous limit.

5. **conditionally**

Its absolute series is harmonic.

6. **False**

Rational decimal truncations of $\sqrt{2}$ are Cauchy, but their unique real limit is irrational. The equivalence between Cauchy and convergent depends on completeness of the ambient space.

7. **False**

The terms $1/n$ tend to zero, but the harmonic series diverges. Tending to zero is necessary, not sufficient.

8. **0.2**

The derivative is $5e^{-(5x)}(1 - 5x)$, positive before $x = 1/5$ and negative after it. The maximum occurs at $x = 1/5 = 0.2$.

9. **$\sum_{n=1}^\infty x^n/n^2$**

For the first series, $|x^n/n^2| \leq 1/n^2$ and $\sum 1/n^2$ converges. Each other series diverges at $x = 1$, so cannot converge uniformly on the whole interval.

10. **1**

The area is half the base times the height: $(1/2)(2/n)n = 1$ for every n . Pointwise convergence to zero does not make these integrals tend to zero.