

12. A.6 · Congruences, divisibility and arithmetic functions

Starter Quiz ·

GRE Subject Test · GRE Mathematics

Name: _____ Score: _____

1. How many solutions modulo 20 does $8x \equiv 12 \pmod{20}$ have?

2. Find the inverse of 5 modulo 13 (between 1 and 12).

3. Find the least positive x with $x \equiv 1 \pmod{3}$ and $x \equiv 4 \pmod{5}$.

4. What is $7^{222} \pmod{10}$?

5. $\varphi(36)$ equals

☐ 6

☐ 12

☐ 18

☐ 24

6. For integers a and $n \geq 2$, $a^{\varphi(n)} \equiv 1 \pmod{n}$ holds whenever $\gcd(a, n) = 1$.

☐ True ☐ False

7. How many residue classes modulo 6 satisfy $0x \equiv 0 \pmod{6}$?

8. Which congruence is equivalent to $6x \equiv 9 \pmod{15}$ after dividing by the gcd?

☐ $x \equiv 3 \pmod{5}$

☐ $2x \equiv 3 \pmod{5}$

☐ $6x \equiv 3 \pmod{5}$

☐ $2x \equiv 3 \pmod{15}$

9. Since $\varphi(8) = 4$, reducing the exponent proves $2^{10} \equiv 2^2 \pmod{8}$.

☐ True ☐ False

10. What is the least positive integer N such that $18N$ is a perfect cube?

☐ 3

☐ 6

☐ 12

☐ 18

12. A.6 · Congruences, divisibility and arithmetic functions

Answers ·

GRE Subject Test · GRE Mathematics

1. **4**

The gcd is 4 and divides 12, so there are four residues modulo 20. Reducing gives $2x \equiv 3 \pmod{5}$, hence $x \equiv 4 \pmod{5}$. The four lifts are 4, 9, 14 and 19 modulo 20.

2. **8**

We need $5x \equiv 1 \pmod{13}$. $5 \cdot 8 = 40 = 3 \cdot 13 + 1$, so the inverse is 8. (The Euclidean algorithm finds it systematically.)

3. **4**

List numbers $\equiv 4 \pmod{5}$: 4, 9, 14, ... The first, 4, already leaves remainder 1 when divided by 3, so $x = 4$. By the CRT the solution is unique mod 15.

4. **9**

$\varphi(10) = 4$, $222 \equiv 2 \pmod{4}$, $7^2 = 49$.

5. **12**

$36 \cdot \frac{1}{2} \cdot \frac{2}{3}$.

6. **True**

True. Euler's theorem requires coprimality. For example, 2 and 8 are not coprime, and $2^4 \equiv 0 \pmod{8}$ rather than 1.

7. **6**

Every residue 0, 1, 2, 3, 4 and 5 works. The gcd is $\gcd(0, 6) = 6$, which divides the target 0.

8. $2x \equiv 3 \pmod{5}$

The gcd is 3, so the coefficient 6, target 9 and modulus 15 all divide by 3. The reduced class modulo 5 lifts to three classes modulo 15.

9. **False**

Euler exponent reduction requires an invertible base. The actual residues are $2^{10} \equiv 0 \pmod{8}$ and $2^2 \equiv 4 \pmod{8}$, so they differ.

10. **12**

Factor $18 = 2 \cdot 3^2$. To make all product exponents multiples of 3, add two factors of 2 and one factor of 3. Thus $N = 2^2 \cdot 3 = 12$ and $18 \cdot 12 = 216 = 6^3$.