1. How many units does $\mathbb{Z}/12\mathbb{Z}$ have?

   _____

2. Which quotient ring is a field?

   ☐ $\mathbb{Z}/7\mathbb{Z}$

   ☐ $\mathbb{Z}/10\mathbb{Z}$

   ☐ $\mathbb{Z}/4\mathbb{Z}$

   ☐ $\mathbb{Z}/9\mathbb{Z}$

3. $\mathbb{Z}$ is an ideal of $\mathbb{Q}$.

   ☐ True ☐ False

4. In $\mathbb{Z}/6$, which x satisfy $2x = 2$?

   ☐ 2

   ☐ 0

   ☐ 3

   ☐ 4

   ☐ 5

   ☐ 1

   Tick every correct option.

5. In a Boolean ring, $2a =$ _____ for every a.

   _____

6. Under the definition used in this lesson, zero is a zero divisor.

   ☐ True ☐ False

7. Every finite integral domain is a field.

   ☐ True ☐ False

8. In $\mathbb{R}[x]/(x^2+1)$, which class is the inverse of x?

   ☐ 0

   ☐ x^2

   ☐ x

exercises.lol exercises.com.cn

9. Which ideal is prime but not maximal in the stated ring?

   ☐ $6\mathbb{Z}$ in $\mathbb{Z}$

   ☐ $5\mathbb{Z}$ in $\mathbb{Z}$

   ☐ $2\mathbb{Z}$ in $\mathbb{Z}$

   ☐ (0) in $\mathbb{Z}$

10. In $\mathbb{R}[x]/(x^2+1)$, $x^4 + 2x^2 + 3$ equals which real constant?

    _____

1. **4**

   The units of $\mathbb{Z}/12\mathbb{Z}$ are the residues coprime to 12: 1, 5, 7, 11. That is $\varphi(12) = 4$.

2. $\mathbb{Z}/7\mathbb{Z}$

   $7\mathbb{Z}$ is maximal because 7 is prime.

3. **False**

   $\frac{1}{2} \cdot 1 \notin \mathbb{Z}$.

4. **4; 1**

   $2x \equiv 2 \pmod 6$ means 6 divides $2x - 2$, i.e. $x \equiv 1 \pmod 3$. In $\mathbb{Z}/6$ that gives $x = 1$ and $x = 4$ ($2 \cdot 4 = 8 \equiv 2$). Two solutions appear because 2 is not a unit mod 6.

5. **0**

   From $(a + a)^2 = a + a$.

6. **False**

   A zero divisor must itself be nonzero and kill another nonzero element. Zero is excluded by this convention.

7. **True**

   Multiplication by a nonzero element is injective because cancellation holds. On a finite set it is therefore onto, so 1 has a preimage and the element has an inverse.

8. $-x$

   The relation is $x^2 = -1$, so $x(-x) = -x^2 = 1$. The class x itself squares to -1 rather than 1.

9. (0) **in** $\mathbb{Z}$

   $\mathbb{Z}/(0)$ is $\mathbb{Z}$, a domain but not a field. The quotients by $2\mathbb{Z}$ and $5\mathbb{Z}$ are fields, while $\mathbb{Z}/6\mathbb{Z}$ has zero divisors.

10. **2**

    Use $x^2 = -1$ and $x^4 = 1$. The expression is $1 - 2 + 3 = 2$.

exercises.lol exercises.com.cn