1. What is the order of 8 in the additive group $\mathbb{Z}/20\mathbb{Z}$?

   _____

2. A subgroup of order 6 sits inside a group of order 42. What is its index?

   _____

3. What is the order of $(1\,2\,3)(4\,5\,6\,7)$ in S_7?

   _____

4. How many conjugacy classes does S_4 have?

   ☐ 24

   ☐ 5

   ☐ 4

   ☐ 6

5. A group of order 12 need not have a subgroup of every order dividing 12.

   ☐ True ☐ False

6. With rightmost-first composition, $H = \{e, (1\,2)\}$ and $g = (1\,2\,3)$ in S_3. Which is gH?

   ☐ $\{(1\,2\,3), (2\,3)\}$

   ☐ $\{e, (1\,2)\}$

   ☐ $\{(1\,2\,3), (1\,3)\}$

   ☐ S_3

7. Every abelian subgroup is normal in every group containing it.

   ☐ True ☐ False

8. For $\varphi : \mathbb{Z}/6\mathbb{Z} \to \mathbb{Z}/6\mathbb{Z}$, $\varphi(x) = 2x$, how many elements does $(\mathbb{Z}/6\mathbb{Z})/\ker\varphi$ have?

   _____

9. How many 3-cycles are in A_4?

   _____

10. Which coset set fails to become a group under the usual coset multiplication?

    ☐ $(\mathbb{Z}/12\mathbb{Z})/\{0, 3, 6, 9\}$ (additive)

    ☐ $S_3/\{e\}$

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☐ $S_3/\{e, (1\,2)\}$

1. **5**

   In $\mathbb{Z}/n\mathbb{Z}$ the order of a is n / gcd(a, n) = 20 / gcd(8, 20) = 20 / 4 = 5. Check: $5 \cdot 8 = 40 \equiv 0$.

2. **7**

   By Lagrange's theorem the index is |G| / |H| = 42 / 6 = 7, the number of cosets.

3. **12**

   The cycles are disjoint, so the order is the lcm of their lengths: lcm(3, 4) = 12.

4. **5**

   Conjugacy classes in S_n correspond to cycle types, i.e. partitions of n. The partitions of 4 are 4, 3+1, 2+2, 2+1+1 and 1+1+1+1, so there are 5 classes.

5. **True**

   True. The converse of Lagrange's theorem fails: A_4 has order 12 but no subgroup of order 6.

6. $\{(1\,2\,3), (1\,3)\}$

   Multiply g by each element of H: ge = (1 2 3) and g(1 2) = (1 3). The other two-element set listed is Hg.

7. **False**

   The subgroup {e, (1 2)} is abelian but not normal in S_3. The valid rule requires the ambient group to be abelian.

8. **3**

   The kernel is {0, 3}, so the index is 6/2 = 3. The image {0, 2, 4} also has three elements; the six-element codomain is not the image.

9. **8**

   Choose three of the four symbols in 4 ways. Each chosen triple has two cyclic orientations, giving $4 \times 2 = 8$. All 3-cycles are even.

10. $S_3/\{e, (1\,2)\}$

    Only {e, (1 2)} in S_3 is not normal. A_3 has index 2, every subgroup of the additive abelian clock group is normal, and the identity subgroup is always normal.

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